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A COMPONENT CURVE ANALYSIS OF DATA FROM COMPUTER ASSISTED DRILL IN ARITHMETIC

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Component Curve Analysis of Data From Computer Assisted Drill in Arithmetic": submitted by Robert F. Mullen in partial fulfillment of the requirements for the degree of Master of Education.

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ABSTRACT

The technique of component curve analysis as developed by Ledyard Tucker was applied to data from computer-assisted drill in arithmetic. Twenty-one second grade students participated in seven sessions of drill over the course of two weeks.

A scoring system was devised which weighted answers with response latencies. The component curve analysis of these scores produced three components. The first was found to relate to initial ability in arithmetic as measured by a pre-test, to IQ, to the score on the Gates-MacGinitie Reading Test, and to performance on a digit span test. This first component was interpreted as general ability in arithmetic. The second component was related to attention as measured on a second computer task designed to yield a measure of the level of attention at various points throughout the drill sessions. The third component could not be adequately identified.

A discussion of some of the outstanding problems with the technique of component curve analysis was included in the paper. It was concluded that the results from the limited set of data of this study indicate that the technique has great potential in the investigation of individual differences in learning.

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CHAPTER I

INTRODUCTION

This paper presents an analysis of the data from seven sessions of computer-assisted drill in arithmetic. Twenty-one second grade students worked thirty minutes a day on simple mathematical expressions of the form $a + b = c$. The purpose of this paper is not to extract results of global significance from these limited data. Rather the purpose is to provide an example of a method of data analysis which appears to have considerable promise as a means of identifying individual parameters of performance in learning tasks.

Whenever learning data are analyzed, there is a great temptation to employ curves of some sort to illustrate what has taken place. With data from computer-assisted instruction, the temptation is even greater for the computer itself can perform what calculations are necessary and then display the curves. Unfortunately, many types of learning curves have been shown to give a distorted picture of the learning process. Perhaps the most vital criticism of traditional learning curves (which are based on averages of one kind or another) is that they obscure individual differences and, in fact, may not represent any individual very well. If the purpose in using computer-assisted drill is to provide individual instruction, then any method of analysis which obscures individual differences has limited value. Still the mass of data which the computer can process and display must be reduced to a manageable subset.

In 1960, Ledyard Tucker introduced the method of component curve analysis which gives promise of resolving some of these problems. In effect the method calls for a component analysis of the learning data to determine the significant components. The components are easily interpretable as curves. Thus instead of using one average curve to describe the data, as many curves as may be necessary to account for the individual differences are employed. A score for an individual becomes a weighted sum of the various component curves. If the components can be identified by their correlations with independent reference variables, then it is possible to identify what underlies each individual's performance. For instance, if one of the components extracted from the arithmetic drill data is seen to represent numerical aptitude and another component the level of attention, then an examination of the individual weights for each student on each of the components could indicate which low scores were the result of inability and which of boredom.

This paper then, examines in detail the method of component curve analysis and applies the method to the data from an arithmetic drill. The results obtained are not generalized beyond the group of students who participated in the study. The purpose of the study is to examine the feasibility of this method of data analysis with a small sample. If the method of component curve analysis is seen to yield meaningful results from a small sample over a short period of time, then it can reasonably be expected to yield more stable and useful results from more comprehensive data.

Chapter II gives a review of some of the research that has been

done in computer-assisted drill in arithmetic. A summary of the criticisms of traditional learning curves is included followed by a description of the method of component curve analysis and a brief summary of some of the unresolved problems with the technique.

A detailed description of the design of the study follows in Chapter III. The scoring system is illustrated and the reference variables, which are used in an attempt to identify the components, are explained.

Chapter IV outlines the exact method of analysis used and gives the preliminary results, i.e. the number of components chosen for each of the task variables.

A summary of the results appears in Chapter V and a discussion of the results and their significance follows in Chapter VI.

CHAPTER II

ANALYSIS OF INDIVIDUALIZED INSTRUCTION

Arithmetic Instruction Using Computers

A great deal of work has been done recently in the field of computer-assisted instruction. Research has been reported from many universities, including the University of Alberta, Florida State University, the University of Illinois, Pennsylvania State University, Pittsburgh University and Stanford University.

Suppes and Morningstar (1969) have reported on the program at Stanford in which computer-assisted drill in arithmetic was used to supplement the classroom teaching of grade school arithmetic. Suppes and Ihrke (1969) have described in detail the results of the program and Suppes and Jerman (1968) provided an outline of the curriculum. While the results of the Stanford program have been mixed, the study has shown that students, especially those who are regarded as slow or deprived, do benefit from drill on the computer in addition to traditional instruction.

The advantages of computer drill for the student are many. A drill session can be individualized so that the student begins, works, and finishes independent of other students. An incorrect response can be corrected immediately. Missed sessions do not leave the student a day behind the rest of the class. Of more concern for the present study, however, is an important advantage for the researcher. With little cost in time or effort, complete records including data on the response latencies can be kept for all drill sessions.

Having available the response latency for each problem worked on

by the student opens up many possible areas of research. One of these possibilities has been explored by Suppes and Groen (1967) who introduced five models which differentially predict response latency for the addition of two one-digit numbers. Latency times were then used to determine which of the models most accurately described the actual numerical operations performed by the students in solving the simple addition problems. Suppes (1967) and Suppes, Hyman, and Jerman (1967) used regression techniques to predict latencies for more complicated arithmetic problems.

The present study is similar to the work done at Stanford in that computer-assisted drill is used to supplement classroom instruction in arithmetic. However, where Suppes and his associates are primarily concerned with generating general models of performance, the purpose of this study was to investigate the individual differences that emerge during the drill and to examine the utility of a technique for identifying individual differences in performance.

Learning Curves

The use of curves to give a pictorial representation of learning data is by no means a recent development. Nor is criticism of the use of learning curves. Most of the criticism has been aimed at the various types of average learning curves.

The average learning curve, in many instances, has been shown to obscure individual differences. Hilgard (1938) pointed out that the Vincent curve procedure, which averages individual performance curves, may reveal the form of the learning function as learning reaches a

criterion of mastery, but nothing is revealed about individual differences. Hilgard discussed several methods of weighting individual curves which help to more accurately reveal the learning function.

Sidman (1952) looked at a frequently used learning function,

$$y = M - M\,e^{-kx}.$$

If different values are taken for M and k and individual curves are constructed, all will be of the same form. But if the individual curves are averaged, the average curve will be of a different form. Furthermore, a number of other curves can be found which approximate the form of the individual curves better than the average curve.

Where the individuals' curves are of different forms, Hayes (1953) pointed out, the form of the average curve depends on the distribution of the individual curves. Hayes showed that if the only difference between individuals is in their speed, that is if the individual curves are of similar form but transposed horizontally, then the backward learning curve gives a true picture of the learning function. Otherwise, Hayes stated, individual curves should be used rather than any type of average curve.

Estes (1956) took a more analytic approach to the problem of average curves. He pointed out that an average curve could be derived from any of an infinite number of combinations of individual curves, and that it is, in general, not possible to deduce anything about the individual curves from the average curve. To show the relation between individual and average curves, he defined three classes of function or curve.

Class A includes those curves whose form is unmodified by averaging. Further, the parameters of the mean curve are the means of the corresponding parameters for the individual curves. Such functions as $y = a + bx$, $y = a \log x$, and $y = a \sin x + b \cos x$ are in this class. Average learning curves are valid descriptions of individual curves for this class.

In Class B are those curves for which averaging leaves the form unchanged but complicates the interpretation of the parameters. For these curves, the parameters of the average curve are functions of the parameters of the individual curves. Functions of the form $y = \log bx$ and $y = \frac{1}{a} + \frac{b}{ax}$ are in this class. Average curves are useful for this class as well, but care must be taken in interpreting the parameters.

Estes' Class C is made up of those curves whose form is changed by averaging. The function mentioned above, $y = M - M e^{-kx}$, is an example. It is for this class of curves that averaging is least useful.

It should be noted that most traditional learning curves would be classified as Class C functions.

There are additional problems with using single curves to represent learning. A single curve suggests there is a single factor or component underlying learning. While this may be the case for some simple laboratory tasks, it is far from universally accepted that all learning depends on a single factor. It is possible, of course, to postulate more than one factor making up an average curve. Stake (1961), using a modified version of the Thurstone learning curve, found up to ten para-

meters which could be investigated including the degree of curvature and the coordinates of the point of intersection of the initial and final asymptotes. The choice of the parameters to use is arbitrary and, Stake pointed out, could depend on the particular learning situation.

The use of a single curve may fail to represent learning for yet another reason. Bakrick, Fitts, and Briggs (1957) showed that varying the criterion for success in a dichotomous task results in entirely different curves. Any learning curve has two areas of insensitivity, the initial area where improvement has not yet been great enough to register and the final asymtotic region where there is little room left for improvement. In the central portion where the error rate is around fifty per-cent the curve is most sensitive to learning. Therefore varying criterion for success could produce curves which are relatively insensitive to the actual learning which is occurring. This can be the case either when the criterion is too difficult or when it is too easy. This characteristic is particularly important when two curves are to be compared.

What was needed then was a method which would preserve the form of the individual curves while simplifying the data and which would provide information about the factors underlying learning.

Component Curve Analysis

Tucker (1958) showed that factor analytic methods can be applied to a non-linear functional relationship to determine its parameters. This led Tucker (1960) to apply the method to learning data and develop the procedures for component curve analysis. Instead of investigating

either general attributes of learning <u>or</u> individual differences, the researcher, using Tucker's procedure, could now look at both phenomena at once.

The classical factor analytic model is generally formulated as

$$(2.1) \quad z_{ji} = a_{j1}F_{1i} + a_{j2}F_{2i} + \ldots + a_{jm}F_{mi} + d_jV_{ji}$$

$$(i = 1,2, \ldots N;\ j = 1,2, \ldots n)$$

where z_{ji} is the standard score for an individual i on a test j; a_{jk} is the factor loading of the test j on the common factor k; F_{ki} is the factor score of individual i on factor k; d_j is the loading of test j on the unique factor; and V_{ji} is the factor score for individual i on the unique factor for test j (Harman, 1968). This model postulates a linear relationship. The usual procedure is to solve for the common factor loadings and disgard the unique factors as well as those of the common factors which do not contribute significantly to the variance of the original matrix Z.

The basic equation of component curve analysis is similar to equation (2.1). Using Tucker's (1966) notation,

$$(2.2) \quad x_{ji} = b_{j1}y_{1i} + b_{j2}y_{2i} + \ldots + b_{jn}y_{ni}$$

$$(i = 1,2, \ldots N;\ j = 1,2, \ldots, n;\ n < N)$$

where x_{ji} is the observed score for individual i on trial j, and for component k, b_{jk} is a coefficient dependent on the trial and y_{ki} is an individual parameter. The interpretation of the terms, however, is quite different from that in classical factor analysis. The z_{ji} of equation (2.1) is said to be a weighted sum of the factor scores for an individual where the weights are the factor loadings for the variable.

In contrast, the x_{ji} of equation (2.2) is considered to be a weighted sum of reference learning curves where now the weights are the individual factor or component scores y_{ki} and the reference learning curves are the component loadings b_{jk}. Component curve analysis is thus analogous to Fourier Analysis in that a complicated function or curve is broken down into a weighted sum of more simple functions or curves. An initially non-linear relationship is transformed into a linear relationship similar to that of factor analysis. There is no reason to expect, of course, that the component curves will be linear, or quadratic, or in any simple, readily identifiable form as they are in Fourier Analysis. What is claimed for component curve analysis is that the component curves will represent significant parameters in learning which might possibly be identified.

The solution of equation (2.2) is similar to the principal axes solution of the classical factor analysis equation. In matrix form, equation (2.2) becomes,

(2.3) $X = B\ Y$

where X is the n x N matrix of the scores of N people over n trials, B is an n x n matrix of component curves, and Y is an n x N matrix of component scores. (It is assumed here that $n < N$.)

As in factor analysis, however, the interest is seldom in reproducing the entire input matrix. Instead of using all the terms in equation (2.2), an approximation is made to X using a limited number of components, r. Equation (2.2) becomes

$$(2.4)\quad \hat{x}_{ji} = b_{j1}y_{1i} + b_{j2}y_{2i} + \dots + b_{jr}y_{ri}$$

$$(i = 1,2, \dots N;\ j = 1,2, \dots, n;\ r < n < N)$$

or in matrix notation,

$$(2.5)\quad \hat{X}_r = B_r\ Y_r$$

where $\hat{X}_r$ is an n x N matrix which is a least squares approximation to X employing r components. B_r is now an n x r matrix, and Y_r is an r x N matrix.

Equation (2.5) is solved using a technique derived by Eckart and Young (1936). The technique consists of dividing the original matrix into a product of three new matrices,

$$(2.6)\quad X = VGW$$

where V is an n x n orthogonal matrix (whose columns are called the left principal vectors of X); W is an N x N orthogonal matrix (whose rows are called the right principal vectors of X); and G is an n x N matric whose upper left section contains the principal roots, λ_1, λ_2, ..., λ_n, in the diagonal and which contains zeros elsewhere. The approximation matrix $\hat{X}_r$ is formed by using only the first r columns of V, the first r principal roots, and the first r rows of W. Equation (2.6) becomes

$$(2.7)\quad \hat{X}_r = V_r\ G_r\ W_r$$

where V is an n x r matrix, G an r x r diagonal matrix, and W an r x N matrix.

Tucker defined B^*_r and Y^*_r two possible values for B_r and Y_r as follows

$$(2.8)\quad B^*_r = (N)^{-\frac{1}{2}}\ V_r\ G_r$$

$$(2.9)\quad Y^*_r = (N)^{\frac{1}{2}}\ W_r$$

The value $(N)^{-\frac{1}{2}}$ is used to scale B_r^* so that the simple sum of the component curves will be of the same order of magnitude as the weighted sum. This makes it possible to compare curves derived from different groups. Weitzman (1963) explained in detail the rationale behind the scaling. The value $(N)^{\frac{1}{2}}$ in the expression for Y_r^* is necessary to preserve the equality of equation (2.5). The B_r and Y_r of equation (2.5) are obtained from B_r^* and Y_r^* by rotation. The possible criteria for this rotation will be discussed later.

The solution of equation (2.6) is carried out in the following manner. If the original matrix is post-multiplied by its transpose and divided by N to form the mean cross product matrix, then

(2.10) $XX'/N = V\,H^2 V'$

where the characteristic roots in H^2 are the scaled (by N) squares of the principal roots, $\lambda_1, \lambda_2, \ldots \lambda_n$. The characteristic vector matrix V contains left principal vectors of equation (2.6).

It does not matter where in the analysis the extra terms corresponding to $\lambda_{r+1}, \lambda_{r+2}, \ldots, \lambda_n$ are dropped. Perhaps the best approach is to carry all of the components until the solution is complete. At this point, the final columns of B and the final rows of Y can be dropped to yield B_r^* and Y_r^*. If the complete V and G matrices are used,

(2.11) $B^* = VH$

and Y* can be obtained from

(2.12) $Y^* = H^{-2}\,B'\,X.$

If the last components are dropped from V and G immediately,

(2.13) $Y_r^* = H_r^{-2}\,(B_r^*)'\,X.$

The above analysis is parallel to that of the principal axis solution but the starting points are different. Factor analysis begins with a correlation matrix while component curve analysis begins with the cross product or variance/covariance matrix. It is this difference as well as the difference in the input data (scores on tests versus repeated scores on trials) which accounts for the different interpretation of the results.

Problems in Component Curve Analysis

Thus far, the analysis described has been relatively straightforward. Those who have used the technique do not differ markedly in the method of analysis described above. Ross (1964) divided the characteristic roots among B and Y, but this does not affect the actual curves obtained other than to scale them up or down. Yet there are many areas left to contention. Among the problems still unsolved are the number of components to keep, the significance of the data means, and the value of various types of rotation.

Tucker (1966) gave two methods for deciding on the number of significant components. The first consists of forming a mean square ratio for each factor k using

$$(2.14) \quad (MSR)_k = \lambda^2_k \, (DF)_k / (dF)_k \sum_{p=k+1}^{n} \lambda^2_p$$

where

$$(2.15) \quad (dF)_k = n + N + 1 - 2k$$

$$(2.16) \quad (DF)_k = (n - k)(N - k)$$

This is a ratio of each characteristic root (of XX') to the total of the remaining values. In other words, it is the ratio of the variance taken

out by component k to the variance remaining. The distribution is similar to an F distribution for very large samples. Tucker, however, gave no distribution for small samples. Weitzman (1959) used a similar ratio of variances.

Tucker's second method is based on the assumption that the series of coefficients over trials for any valid component should form a smooth curve. The differences between consecutive values of the left principal vectors (columns of V) are squared and used as an index of the smoothness of each curve. Once again, the values of this first difference which represent meaningful factors must be chosen arbitrarily.

Gulliksen (1959) in discussing learning curves suggested an approach similar to Tucker's mean square ratio. He advocated looking at the difference between consecutive characteristic roots. If the difference between λ_k and λ_{k+1} is significant and the difference between λ_{k+1} and λ_{k+2} is insignificant, then there are k valid components.

Weitzman (1963) suggested using the runs test. Since the first component will account for the maximum variance possible and approaches the mean curve, the remaining components will fluctuate about the zero line. If a component does not exhibit significant trend, it is regarded as representing error or noise. It and all succeeding components are disgarded. Siegal (1956) gave tables which can be used to determine significant trends. Unfortunately, the number of trials must be eight or greater for the runs test to have any meaning.

A discussion of these methods follows in Chapter IV. An attempt is made to derive a statistical test to determine the number of com-

ponents which should be retained.

The second problem has to do with the means of the data. Ross (1963) showed that removing person or test means before performing a factor analysis produced a shift of the origin to the centroid of the tests or persons respectively. In component curve analysis, the first component closely approximates the means of the data. It is possible to perform a rotation so that the first curve will pass through the data means. This tempts one to remove the means in advance. But Ross (1964) showed that removing the test means in advance did not simply remove the first component. Instead, similar to the results in factor analysis, removing the means shifts the origin to the centroid of the person configuration while preserving the number of components. This covariance solution obtained by removing the trial means from the data has many desirable properties but some information is lost when the means are removed. The problem is discussed more fully in Chapter IV.

Finally there is the question of rotations. Mathematically there is no difficulty. The rotated trial loadings and the rotated component scores are given by

$$B_r = B_r^* T \tag{2.18}$$

$$Y_r = T^{-1} Y_r^* \tag{2.19}$$

where T is any non-singular r x r matrix. The problem is in choosing the criterion to use in deriving T. The factor analysis answer of simple structure is not generally applicable. There is no reason to expect simple structure in the trial space, i.e. a trial loading on one component but not on another. It is conceivable that in some applications,

we could expect an approximation to simple structure in the person space, i.e. individuals loading more heavily on one curve than on another.

Tucker (1966) used as his criterion the requirement that all of the component curves have non-negative entries, non-negative slopes when smoothed, and reach an asymptote. Ross (1964) showed that putting one of the components through the centroid of the person configuration results in a corresponding curve which is proportional to the original data means.

Duncanson (1964) used the unrotated component scores as test scores in a factor analysis. The traditional factor analytic rotations were then used on the factors obtained.

In general, the rotations employed will depend on the exact nature of the learning task which produced the components. Cliff (1962) provided a technique for rotating to various types of functional relationships. Thus if there is some apriori knowledge about the components, a rotation can be derived which will attempt to fit the obtained components to the theoretical components.

The rotations employed in the present study and the rationale for their use are discussed in Chapters IV an V.

CHAPTER III

DESIGN OF THE STUDY

The Arithmetic Task

The learning task was a drill in basic arithmetic. Expressions of the form a + b = c were presented. In each case, one of the values was missing. The student was asked to supply the missing number.

A drill session consisted of sixty problems, ten of each of the types shown below.

Type I: a + b = ___

Type II: a - b = ___

Type III: a + ___ = c

Type IV: a - ___ = c

Type V: ___ + b = c

Type VI: ___ - b = c

The problems were presented in the order listed, ten of Type I, ten of Type II, etc. The values in the expressions were generated randomly as each problem was presented. For addition, "a" and "b" were numbers from 0 to 9 and for subtraction, "b" and "c" ran from 0 to 9.

The students performed this task at terminals of the IBM 1500 Computer System at the University of Alberta. Each terminal included a display screen, an audio headset, and a keyboard. Although there were sixteen terminals available, no more than eleven students worked simultaneously. This decreased the time lag between problems and insured that no students would have to wait if one of the terminals were inoperative.

At the first session, an audio message (AA1) was played giving the instructions for the task. (See Appendix A for a script of the audio messages.) The problems were then presented on the display screen (a cathode ray tube.) The numbers were approximately an inch high, six times the size of numbers which would normally appear on the screen. The larger numbers were created so that young children could recognize them easily.

The student answered or solved the expression by pressing a number on the keyboard. If he answered correctly, two forms of reinforcement were given. His answer appeared immediately in the blank position of the arithmetic expression and brief words of encouragement were fed to him through the headset. These audio messages are shown in AA2. If he answered incorrectly, his answer was displayed at the bottom of the screen superimposed with an "X". The correct answer was then displayed in the blank. No audio message was given.

A student was allowed thirty seconds to respond to a problem. If he did not answer in the allotted time, an audio message (AA3) reminded him to respond more quickly. The correct answer was then displayed in the blank.

When a student completed the course, an audio message (AA4) directed his attention back to the screen where a summary of the number of problems answered correctly and the number answered incorrectly appeared.

For a more complete description of the drill program, see Fitzgerald, Maguire, and Mullen (1970).

The students were given seven drill sessions, one per day. The sessions were held in the mornings of five consecutive school days and on the following Monday and Wednesday.

<u>The Attention Task</u>

It seems reasonable to assume that for repeated drill sessions, the level of attention of the student will play some role in accounting for the variance of the scores. Thus a task was programmed for the computer which would provide measures of the level of attention for students at different times in the course of the study.

This attention task required the student to press the space bar as fast as possible whenever a number appeared on the display screen. A simple discrimination was required as letters appeared on the screen with the same frequency as numbers. The stimuli were presented one at a time in varying positions on the screen. The task continued until twenty numbers had appeared on the screen. (The number of letters presented during the task could vary but would be about twenty.) The instructions which the student received are given in the Audio Script in Appendix B. The attention task was given prior to the drill session on the first day. It then began automatically at the termination of the arithmetic drill on each session. A complete session, including both tasks, ran for about thirty minutes. Since each student could work at his own speed, some finished earlier than others. As they completed a session, the students were allowed to leave the terminal room.

The Subjects

The subjects were drawn from the second grade population at the Forest Heights Elementary School in Edmonton. Out of a total of forty-four students, twenty-one were chosen to participate in the drill sessions (experimental group). The remainder of the students were considered a control group and made the trip to the University on one of the days to work at a non-arithmetic task on the computer.

Experimental and control groups were chosen in the following way. The students were ranked according to their level of achievement in mathematics as measured by tests taken in the school as part of an IPI program of math instruction. It was found that the first-ranked and last-ranked students were, respectively, far ahead and far behind the rest of the class. These two students were dropped from the study. The remaining students were paired, the second-ranked with the third-ranked, the fourth with the fifth, etc. One student was chosen randomly from each pair to participate in the study. The experimental group of twenty-one participants was then divided into two subgroups, one of ten students and one of eleven. Students in a subgroup worked simultaneously on the tasks. The subgroups worked in two consecutive periods on the computer. The order in which each worked and waited for the other was varied.

While there was a great deal of variability in the levels of achievement of those chosen to participate in the study, all of the students had some previous exposure to the type of problems presented in the drill sessions. After an initial period of timidity and awe, the students had no difficulty operating the terminals.

A supervisor was present in the room at all times but the atmosphere was relaxed. Any questions the students had were answered. No attempt was made to make them work faster. At the end of a session, the students compared their totals.

Only one of the twenty-one students missed a session because of illness. Two others, however, discovered that by turning off their audio units they could disrupt the program (but only on their own terminals). This recurred often enough so that the data from these two students was incomplete and could not be used in the analysis. The analysis was thus carried out for eighteen students.

Arithmetic Task Variables

A complete performance recording was kept for each student throughout the study. The information was stored on magnetic tape and analyzed on the IBM 360 Computer after each session had been completed. The data available included a list of all the problems given, the students' response to each problem, and the latency of each response.

From these data, four dependent variables corresponding to four scoring methods were defined. In each case, the student was graded over an entire session. There were thus seven scores for each student on each of the four arithmetic task variables.

The first variable (Number Correct) was simply the number of expressions which the student solved correctly. The score on this variable could range from zero to sixty.

The second variable (Mean Latency) was the average of all the response times. Since the student was given only thirty seconds to solve

any single expression, the average latencies could range from zero to thirty.

In addition to these two rather straightforward scoring methods, two methods which combined correctness with response latency were used to produce variables three and four. In each case answers were weighted with the response latencies.

The third variable (Linear Score) incorporated a linear weighting scheme. If the student solved an expression correctly, his score was given by

(3.1) $S_L = 10 - t/3$

where t is the response latency in seconds. For an incorrect answer, the linear score was given by

(3.2) $S_L = -^t/3.$

The student's score was thus lowered by increased response latency regardless of whether or not he solved the expression correctly.

The fourth variable (Quadratic Score) was based on a quadratic weighting system. For a correct answer, the quadratic score was given by

(3.3) $S_Q = (t-30)^2/90$

and for an incorrect answer by

(3.4) $S_Q = (t-30)^2/90 - 10$

where again t is the response latency in seconds.

For both variables, the theoretical range of values was from minus six hundred to plus six hundred. With both the linear and quadratic methods, the faster a student responded, the higher his score. The difference between the two scoring systems is that the quadratic method weights

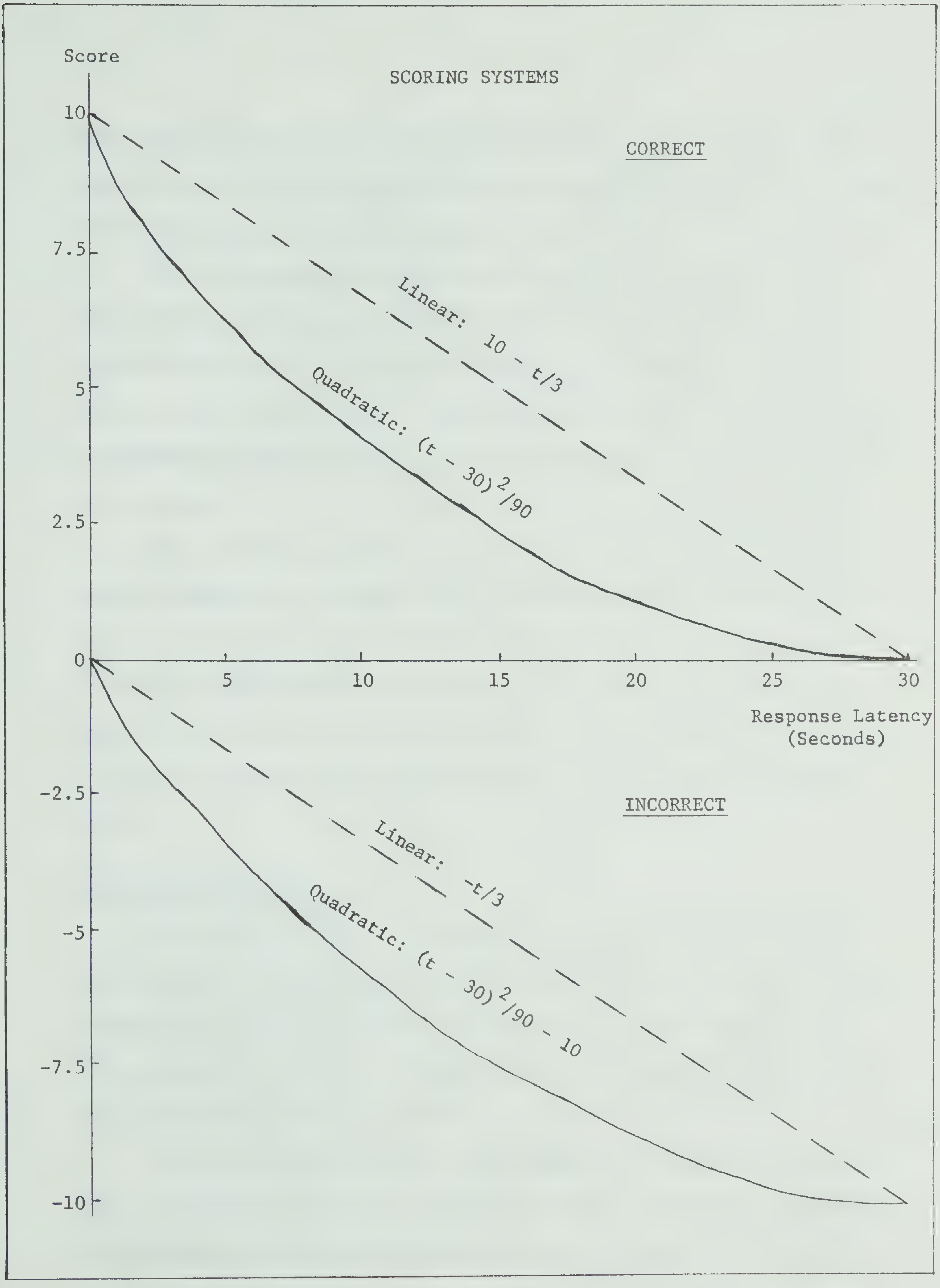
Score
SCORING SYSTEMS
CORRECT
Linear: 10 - t/3
Quadratic: (t - 30)2/90
INCORRECT
Linear: -t/3
Quadratic: (t - 30)2/90 - 10
Response Latency
(Seconds)
10
7.5
5
2.5
0
-2.5
-5
-7.5
-10
5
10
15
20
25
30

FIGURE 3-1

the response latencies more heavily than does the linear method. The quadratic scores were, in general, lower than the linear scores. Figure 3-1 gives a graphical comparison of the two methods.

Each of the four arithmetic task variables was the basis of a component curve analysis. The scores on each arithmetic task variable could be broken down into scores on each of the components found to underlie the original scores. Thus these four dependent variables could be considered compound variables in that they might yield more than one set of scores.

Four different methods of scoring were used so that the results could be compared to determine which scores most efficiently described what had taken place. It would be predicted that the latter two methods of scoring would yield more information, i.e. more component curves, than the first two. If this were the case, then it would be of interest to know how the additional information relates to the reference variables.

Attention Task Variables

The attention task was included in the study because level of attention may be a significant factor in performance on arithmetic drill. By having the students perform the attention task each time they performed the arithmetic task, it was hoped that a dynamic measure of the level of attention would be obtained.

Grim, Kohlberg, and White (1968) measured attention from reaction times on a similar task. They used mean latency, the standard deviation of the latencies, and the difference between the means of the initial

five latencies and the last five. They found these measures to be valid indications of the level of attention for young children. Four variables were defined in this study for the attention task. The purpose once more was to compare the results of each and to determine which variable most efficiently measured not just the level of attention but that part of attention which was related to the student's performance on the arithmetic drill. (These variables too, should be considered compound variables because each yields one or more sets of component curves.)

The first attention task variable (Mean Latency) was the average of the response latencies for all twenty numbers which appeared on the screen. The student was given up to twenty seconds to respond. Thus the range of possible values was from zero to twenty.

The second variable (Difference) was the mean of the last ten response latencies minus the mean of the first ten. The difference could be from minus twenty to plus twenty.

The third variable (Median Latency) was the median response latency. The median ranged from zero to twenty.

The fourth variable (Standard Deviation) was the standard deviation of the response latencies. It's value varied from zero to ten.

(Students usually responded no more than a few times to letters. When they did, an audio message was given repeating the initial instructions. There seemed no reason to involve the incorrect responses in the scoring.)

Reference Variables

Once all of the variables for the arithmetic and attention tasks

have been analyzed, there will be eight groups of component scores. By investigating the correlations between sets of scores, the relationships between the components of the various task variables can be identified. For instance, the scores on the second component derived from the Linear Score variable might be highly correlated with the scores on the first component derived from the Number Correct variable. While this information is valuable, it does not really explain much about the identity of either component. On the other hand, if a set of component scores is correlated with reference variables such as the students' IQ's, then there will be some basis for identifying that component. The function of the reference variables listed below is to aid in identifying the components which are found to underlie the learning task.

Where possible, scores on the reference variables were obtained for the entire population of second graders. Some of the measures, such as IQ, were taken from the school records. Other scores were taken from tests given the students before, during, and after the twelve day period in which they participated in the drill sessions.

In a few cases, complete sets of scores on a reference variable were not available for a student who participated in the drill sessions. Rather than drop the student from the analysis or correlate with missing data, the score was derived either by using regression techniques or merely by giving the student the median score of the rest of the class on that variable. The scores for all of the students on the reference variables are given in Appendix C. Where a score was derived, the method used is indicated.

Ability in Arithmetic

A measure of the student's initial ability to solve the type of problems given during the drill was desired. Thus a pre-test made up of expressions like those to be presented on the computer was given to the students in their classroom a few days before they began the drill. The test was divided into sections, each section containing one of the six types of expressions listed earlier. There was a time limit for each section. The score for this variable was the total number of expressions solved correctly in all of the sections. The test appears in Appendix D. This variable, RV1, is said to represent ability in arithmetic.

Reflection-Impulsivity

Since three of the scoring systems for the arithmetic task take into account the response latency, the scores could reflect in part a predisposition on the part of the student to give answers rapidly or conversely to answer only after careful consideration. Many studies have reported a reflection-impulsivity variable where reflection implies long response latencies and low error rates, and impulsivity implies just the opposite, short response latencies and high error rates. Three measures of this variable were included.

The students were given The Matching Familiar Figures Test during one of the periods when they were waiting to get onto the computer. Kagan (1967) found this test to be the most sensitive for measuring reflection-impulsivity. The test consists of twelve sets of pictures. Each set contains a standard and six similar pictures. The subject was

asked to select the picture in the latter group which was identical to the standard. The response time for his initial selection was recorded. If a selection was incorrect, the student was given additional chances until he picked the correct picture or until he made six mistakes.

The average response time for the selections was RV2. The total number of errors over the twelve items gave a second measure, RV3. In addition, a method for selecting pure groups of impulsives and reflexives was employed, (Kagan, 1965). Those who scored above the median on latency and below the median on errors were classified as reflexives. Those who score below the median on response time and above the median on errors were classified as impulsives. This procedure was followed, based on the entire population of second graders. A value of one was assigned to pure reflexive students, a value of three to pure impulsive students, and a value of two to the others. This variable, RV4, is called the Reflection-Impulsivity Index.

It is likely that arithmetic ability and style of response affect the student's performance on the drill. But there are other variables which produce individual differences in general performance within a group of students. It is not expected that they will be as important as arithmetic ability in identifying the components of this task but it seems reasonable that they might be relevant. Thus the following four variables were included in the analysis.

Vocabulary Score

The students were given the Gates-MacGinitie Reading Test (Primary B, Form 1) prior to the first drill session. The test is divided into

two sections. The first is a vocabulary section in which a picture is shown and four words are given. The student is required to select the word which describes the picture. There are forty-eight items in this section. The student's score on this vocabulary section was called RV5.

Verbal Comprehension

The second section, comprehension, gives a brief paragraph and four pictures. The student selects the picture described by the paragraph. There are thirty-four items. The score on this comprehension section was RV6.

Intelligence

The students had been given the Detroit Beginner's IQ Test earlier in the year. Their scores on this test were taken from the school records for RV7.

Age

The student's age in months was used as RV8.

Short Term Memory for Digits

There is reason to believe that the student's performance on certain short-term memory tasks may be correlated with his ability in mathematics. Thus a reference variable was included which can be thought to measure a more basic ability than is measured by the first reference variable, ability in arithmetic.

A short digit span task was given to the students during the time of the study. A list of numbers was played to the student on a tape

recorder. He could hear the list through only one ear. The student was asked to repeat the list. The task consisted of one span each of five, six, seven, and eight numbers for each ear. The lists were given alternately in the left and right ear.

Since few of the students could repeat lists correctly beyond the five-digit level, a system of scoring was used which recognized partial answers. For a span of length k, one point was given for each correct number in the proper position. The numbers not in the correct position were then scanned and one point was given for each pair of numbers in correct sequence. The total points given was then divided by k to yield a score between zero and one. The value of this variable, RV8, for a student was the sum of his scores on all eight of the spans. The lists used and an example of the scoring system can be found in Appendix E.

It is unlikely that these nine reference variables encompass all of the relevant variables, but it is expected that the components extracted from the arithmetic task variables will correlate with some of them. A list of the reference variables is given in Table 3-1.

Arithmetic Post-test

On the day following the last drill session, an arithmetic post-test was given to the entire class. The test was identical to the arithmetic pre-test except that a seventh section containing all six types of expression was added. The score on this test was used with the pre-test scores in an analysis of covariance to determine whether or not the drill sessions produced learning. The test is included in Appendix D.

TABLE 3-1

REFERENCE VARIABLES

RV1:	Arithmetic Pre-Test
RV2:	Average Response Time on M.F.F.T.
RV3:	Total Errors on M.F.F.T.
RV4:	Reflection-Impulsivity Index
RV5:	Vocabulary Score on G.M.R.T.
RV6:	Comprehension Score on G.M.R.T.
RV7:	IQ (Detroit Beginners)
RV8:	Age (in months)
RV9:	Digit Span Score

CHAPTER IV

METHOD OF ANALYSIS

Before a component curve analysis was performed on the data from the arithmetic drill sessions, a simple test was made to determine if indeed the drill sessions produced learning. This test is described in the first section below. It was decided that if there was evidence of learning, each of the eight task variables described in Chapter III would be analyzed using component curve analysis and the number of significant components for each variable determined. The procedure for determining the number of components is outlined in the second section. An example using one of the arithmetic variables appears in the next section. At this point in the analysis a decision had to be made about the significance of data means. Should the trial means be retained in the data or removed? A method for investigating the effect of the data means is given in the fourth section. Finally, an attempt was made to identify the components utilizing the reference variables and suitable rotations. The procedures that were followed are discussed in the last section.

Analysis of Covariance

One of the basic assumptions underlying the technique of component curve analysis is that the data to be analyzed represents some type of learning. An analysis of covariance was performed using the students' scores on the arithmetic pre-test and post-test to determine whether or not a significant degree of learning took place within the experimental group. A one way analysis of covariance was used rather than an analysis

of variance because adjusting the group means on the dependent variable (score on the arithmetic post-test) through the use of a concomitant variable (scores on the arithmetic pre-test) resulted in a more precise test. See Cox (1958) for a more complete discussion of the rationale for the use of an analysis of covariance.

The hypothesis, of course, was that the scores on the arithmetic post-test for the experimental group, those students who participated in the computer-assisted drill, would be significantly greater than the scores for the control group, those students in the class who did not participate in the drill sessions. Eighteen sets of pre-test and post-test scores were obtained for the drill group and, by coincidence, the same number were obtained for the control group. Thus the analysis was performed with equal n's.

Determining the Number of Components

A great deal of preliminary work was done in an attempt to derive a statistical test for determining the number of components. Much of that work is still in progress. For that reason, only the results which are directly applicable to this study are discussed.

There are two approaches to the number of components problem. The first involves looking at the characteristic roots. Mean square ratios can be formed and compared with the f statistic distributions or the differences between succeeding roots can be compared. The second approach involves looking at the characteristic vectors. First differences can be computed to test for smoothness or the runs test can be used to test for significant trend.

For the type of data in the present study, the second approach is of little use. With a small number of trials, the first difference for a steep, monotonically increasing curve can be larger than the first difference for an obviously random curve. This would lead to rejection of the significant curve or retention of the insignificant one. (For a large number of trials, this possibility is less likely, and the first difference appears to be an adequate measure of "smoothness.") The runs test, as mentioned earlier, is not appropriate for less than eight trials.

The significance of tests on the characteristic vectors depends on the number of trials. For tests on the characteristic roots, a more crucial factor is the number of people. There is no distribution available for mean square ratios for small numbers of people. One reasonable procedure is to look at the ratios and pick out the k ratios that appear to be larger than the rest. The k roots corresponding to these ratios are used in the generation of k components. It would obviously be desirable to find another method which could be used to verify the results obtained from the mean square ratios.

A second possibility* is that succeeding characteristic roots be investigated using the jackknife technique as described by Mosteller and Tukey (1968). With component curve analysis, the first component approximates the data means and the first characteristic root is usually many orders of magnitude larger than the succeeding roots. The remaining roots will usually be of the same order of magnitude. Thus, a plot of

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*This procedure was suggested by Dr. T. Maguire.

the succeeding roots will show a large gap between the first and second roots and small gaps from then on. The usual procedure has been to "eyeball" the gaps to decide which roots after the first are significant. The jackknife technique provides a more accurate measure for looking at the gaps.

The jackknife technique is useful in situations where the sampling distribution of a statistic is unknown. In effect, the technique gives a confidence interval for a statistic by systematically excluding one person at a time from the calculation. For characteristic roots the technique is as follows. A root (say the first) is obtained from the mean cross product matrix XX'/N. This is the square of the root of the N person data matrix X. This "real" value is designated by λ ALL. The first person is then excluded from the data matrix and a new cross product matrix is formed from the N - 1 person data matrix. The first root is computed from the new cross product matrix. The value of this new root is denoted by $\lambda_{(1)}$, indicating that the first person has been omitted from the calculation. The first pseudo-value is defined as

$$\lambda_{*1} = N\ \lambda_{ALL} - (N - 1)\ \lambda_{(1)}.$$

In general, if the ith person has been omitted from the calculation, the ith pseudo-value is

(4.1) $$\lambda_{*i} = N\ \lambda_{ALL} - (N - 1)\ \lambda_{(i)}.$$

An estimate of the true value of the characteristic root, called the best single result, is the mean of the pseudo-values,

(4.2) $$\lambda_{*} = {}^{1}/N\ (\lambda_{*1} + \lambda_{*2} + \dots + \lambda_{*N}).$$

An estimate of the variance is

$$(4.3) \quad S^2_* = S^2/N$$

where

$$(4.4) \quad S^2 = [\sum^n \lambda_{*i}2 - {}^1/N \, (\sum^n \lambda_{*i})^2]/(N - 1).$$

The confidence interval for a characteristic root is then given by

$$(4.5) \quad \lambda_* \pm |t_{n-1}|_{(1 - \alpha/2)} S_*.$$

In actual practice, all n values of λ ALL (the n real characteristic roots) are computed from the mean cross product matrix at one time. Similarly, as each person, i, is dropped from the data matrix, n values of $\lambda_{(i)}$ are computed at once and n values for $\lambda_{*(i)}$ are found (corresponding to the n characteristic roots). After each of N persons has been excluded from the computation, the pseudo-values are used to find the n best single results and the corresponding n confidence intervals.

It is tempting to try and find a rigid statistical rule which could be applied to the confidence intervals to determine the number of significant roots (and thus the number of significant components). Unfortunately, this is not possible. The confidence intervals are set up using the t statistic but the distribution of pseudo-values for any root is at best, skewed to the left and is at times completely unrecognizable. The intervals really have little statistical significance. Further, if more than one comparison is to be made, then the simple t statistic should not be used.

While the intervals cannot be used in a statistical sense, they can be employed to indicate where there are gaps between succeeding roots. For this study, the following procedure was used. Starting with the

first root, the best single result was compared with the confidence interval of the succeeding root. If the best single result was contained in the interval, then the two roots were said to be of the same relative importance. If one was retained, both were retained. The process was carried out until all of the roots had been arranged into groups. These groupings were then used together with the mean square ratios to determine the number of components.

The scores on each component (the rows of the Y matrix) were used to attempt identification of the components. Significant correlations between a set of scores on a component and the set of scores on a reference variable indicate a relationship between the component and the reference variable. As an additional check on the choice of the number of components, the highest order component of those discarded was tested to see if there were any significant correlations with any of the reference variables. If there were significant correlations, that component and any others which had been grouped with it were retained.

After the number of components had been fixed, a measure of the extent of the factoring was computed. If r components are retained, the approximation matrix is given by

(4.6) $\hat{X}_r = B_r Y_r$

as mentioned earlier. $\hat{X}_r$ is a least squares approximation to the data matrix X for r components. The error or residual matrix is given by

(4.7) $X_{Res} = X - \hat{X}_r$.

This error matrix contains the amounts by which the scores computed from r components differ from the original scores. The entries in the error

matrix derived from the data in this study were divided by the original scores and the row means computed to give each person's percentage error. The mean of these error percentages was used as an index of the extent of the factoring.

The procedure discussed in this section is summarized below. The appropriate equation for each step in the analysis is indicated. The analysis begins with the n x N data matrix N.

1) Form the n x n mean cross product matrix, XX'/N.

2) Find the characteristic roots and vectors of the mean cross product matrix. (2.10)

3) Compute the mean square ratio for each of the n characteristic roots. (2.14), (2.15), (2.16)

4) One by one, drop individuals from the original matrix and re-form the mean cross product matrix. Find the characteristic roots and compute the pseudo-values. (4.1)

5) Form confidence intervals for each of the n roots and use the intervals to group the roots. (4.2), (4.3), (4.4), (4.5)

6) Use the mean square ratios and the groupings to determine the number of components, r.

7) Compute the loadings of trials on components. (2.11)

8) Compute the component scores. (2.12)

9) Test the (r + 1) st set of component scores for significant correlations with the reference variables. If necessary, increase the number of components retained.

10) Form the approximation matrix using r components. (4.6)

11) Compute the residual matrix, the individual error percentages, and the overall mean error percentage. (4.7)

The analysis described above was carried out for each of four arithmetic task variables and, to a limited extent, for the four attention task variables. The analysis is presented in detail in this paper for the first arithmetic variable, Number Correct, and the results for the other task variables are summarized.

Preliminary Results

The data matrix for the first arithmetic task variable is given in Table 4-1. The entries are the number of problems solved correctly by each person on each trial. The mean cross product matrix, its roots and vectors, and the mean square ratios are given in Table 4-2. A check of the mean square ratios showed only one ratio which was obviously larger than the rest. Comparing the ratios to an F ($\alpha = .5$) distribution, however, indicated that the second value was also significant.

The results of the jackknife procedure are shown in Figure 4-1. The confidence levels are based on $t_{.975}$ for seventeen degrees of freedom. The first pseudo-value (not shown on the graph) was much greater than any of the succeeding values. There were significant gaps between the second and third pseudo-values and between the fifth and sixth pseudo-values. The roots were grouped as indicated. The decision, based on these groupings, was to retain either one, two, five, or seven components. It should be noted that the first two roots were relatively stable in that the best single result for each was close to the actual value.

The trial loadings and the component scores were then computed. The two matrices are given in Table 4-3. The first three rows of Y (columns of the Y' matrix shown in Table 4-3) were compared with the

TABLE 4-1

ORIGINAL SCORE MATRIX
X'

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	33.0	42.0	36.0	45.0	45.0	47.0	34.0
2	42.0	48.0	55.0	55.0	57.0	56.0	56.0
3	57.0	57.0	56.0	58.0	60.0	59.0	58.0
4	36.0	34.0	33.0	34.0	42.0	37.0	35.0
5	39.0	47.0	51.0	57.0	52.0	57.0	57.0
6	42.0	50.0	49.0	55.0	48.0	49.0	50.0
7	42.0	50.0	51.0	47.0	50.0	52.0	45.0
8	57.0	60.0	58.0	59.0	57.0	59.0	57.0
9	48.0	55.0	56.0	58.0	55.0	58.0	59.0
10	42.0	52.0	51.0	49.0	56.0	58.0	58.0
11	50.0	48.0	58.0	57.0	59.0	55.0	59.0
12	54.0	53.0	57.0	51.0	53.0	56.0	52.0
13	41.0	50.0	55.0	50.0	49.0	45.0	50.0
14	45.0	38.0	51.0	53.0	49.0	54.0	50.0
15	45.0	57.0	56.0	57.0	60.0	54.0	58.0
16	53.0	53.0	59.0	51.0	59.0	57.0	58.0
17	53.0	56.0	55.0	57.0	59.0	56.0	60.0
18	36.0	41.0	42.0	39.0	45.0	44.0	46.0
TRIAL MEANS	45.3	49.5	51.6	51.8	53.1	52.9	52.3
STD. DEV.	7.2	6.8	7.3	6.8	5.6	6.0	7.7

CHARACTERISTIC ROOTS

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

BEST SINGLE RESULT	STANDARD DEVIATION	INTERVAL $t = 2.110$
18458.6	992.9	$16363.5 \le \lambda_1 \le 20553.7$
19.0	6.2	$6.0 \le \lambda_2 \le 32.1$
7.1	2.2	$2.4 \le \lambda_3 \le 11.8$
7.2	2.4	$2.1 \le \lambda_4 \le 12.2$
18.3	5.2	$7.3 \le \lambda_5 \le 29.4$
5.3	1.3	$2.6 \le \lambda_6 \le 7.9$
9.6	1.9	$5.6 \le \lambda_7 \le 13.6$

COMPONENTS

Group 1:	1
Group 2:	2
Group 3:	3, 4, 5
Group 4:	6, 7

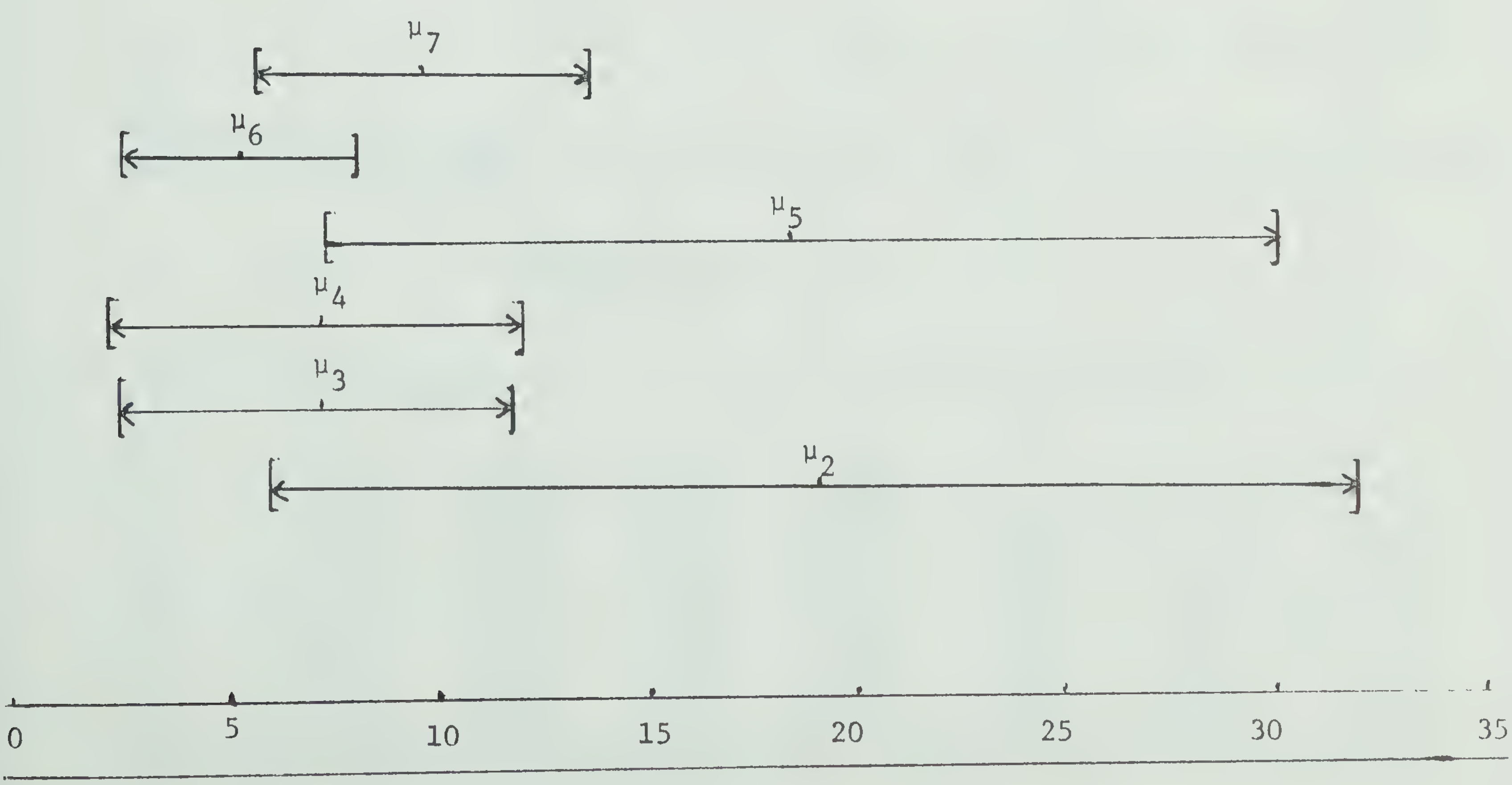


FIGURE 4-1

TABLE 4-2

CHARACTERISTIC VALUES AND VECTORS

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

MEAN CROSS PRODUCT MATRIX

XX'/N

	1	2	3	4	5	6	7
1	2101	2277	2378	2376	2433	2427	2408
2	2277	2496	2593	2596	2656	2650	2629
3	2378	2593	2716	2712	2772	2767	2750
4	2376	2596	2712	2726	2776	2774	2752
5	2433	2656	2772	2776	2846	2837	2815
6	2427	2650	2767	2774	2837	2838	2809
7	2408	2629	2750	2752	2815	2809	2798

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	18462.027	24	102	1245.917*
2	20.397	22	80	1.742*
3	13.257	20	60	1.356
4	11.087	18	42	1.418
5	9.373	16	26	1.717
6	4.874	14	12	1.044
7	4.003	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS

V

	1	2	3	4	5	6	7
1	0.336	0.781	-0.278	0.334	-0.117	0.256	0.095
2	0.367	0.282	0.700	-0.503	0.027	0.011	0.203
3	0.383	0.049	-0.370	-0.330	-0.306	-0.680	-0.223
4	0.384	-0.385	0.206	0.249	-0.682	0.321	-0.179
5	0.392	-0.055	0.064	0.089	0.550	0.126	-0.716
6	0.392	-0.228	0.150	0.556	0.279	-0.422	0.455
7	0.389	-0.324	-0.476	-0.384	0.214	0.418	0.386

scores on the reference variables. The correlations are given in Table 4-4.

With an N of 18, a correlation of approximately .466 is necessary for the .05 level of significance. The second set of component scores had a high (but not significant) correlation with the digit span scores. Together with the results of the jackknife and the mean square ratios, this seemed to indicate that there were two meaningful components.

The first two component curves are graphed in Figure 4-2. The approximation matrix using two components is given in Table 4-5 and the residual matrix appears in Table 4-6. The error percentages and the mean percentage error are given below.

Person	1	2	3	4	5	6	7
	10.43	1.87	1.80	5.28	1.89	4.09	4.47

8	9	10	11	12	13	14	15	16
2.37	1.72	5.05	4.20	2.46	5.49	7.19	3.45	3.23

17	18	Mean
2.32	3.43	3.93

The mean error was less than four percent. Thus it appeared that the scores could be very well reproduced using only two components.

The same procedure was used for the remaining three arithmetic task variables. The original data matrices and the results are given in Appendix F. A summary of the number of components for each task variable appears in Table 4-7.

With the attention task variables, the main concern was not to discover and identify all of the significant components but to find components which related to those found for the arithmetic task variables.

TABLE 4-3

LOADINGS AND SCORES

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

TRIAL LOADINGS
B

TRIAL	COMPONENT 1	2	3	4	5	6	7
1	45.675	3.527	-1.010	1.113	-0.359	0.566	0.191
2	49.857	1.273	2.550	-1.675	0.084	0.025	0.407
3	52.057	0.222	-1.346	-1.098	-0.936	-1.501	-0.446
4	52.128	-1.737	0.749	0.828	-2.089	0.710	-0.357
5	53.303	-0.249	0.235	0.295	1.685	0.279	-1.432
6	53.212	-1.031	0.547	1.851	0.854	-0.932	0.910
7	52.821	-1.465	-1.734	-1.279	0.655	0.923	0.772

COMPONENT SCORES
Y'

PERSON	COMPONENT 1	2	3	4	5	6	7
1	0.786	-0.482	2.740	1.886	0.235	-0.458	-1.048
2	1.030	-1.378	-0.449	0.032	0.331	-0.644	-1.024
3	1.126	1.200	0.122	0.653	0.020	1.250	0.221
4	0.698	0.912	0.060	1.006	1.418	1.101	-1.408
5	1.006	-2.235	-0.071	0.347	-0.347	0.238	1.208
6	0.955	-0.424	0.882	-0.392	-1.727	0.901	0.319
7	0.938	0.462	1.039	-0.057	0.140	-2.285	-0.188
8	1.131	1.433	0.631	0.112	-0.984	0.434	1.093
9	1.084	-0.441	0.163	-0.364	-0.572	0.289	1.342
10	1.021	-0.894	0.191	-0.547	2.246	-0.326	1.562
11	1.075	-0.322	-1.648	0.228	-0.243	0.526	-1.520
12	1.045	1.707	-0.379	0.339	-0.391	-1.409	0.849
13	0.947	0.084	-0.081	-2.102	-1.358	-0.968	-1.218
14	0.948	-0.727	-1.747	2.236	-1.068	-0.618	-0.022
15	1.079	-0.537	0.774	-1.461	0.248	0.667	-1.399
16	1.085	1.001	-1.143	-0.325	1.036	-0.853	-0.183
17	1.102	0.541	-0.123	-0.331	0.174	1.838	0.191
18	0.817	-0.160	-0.322	-0.589	1.413	0.011	0.493

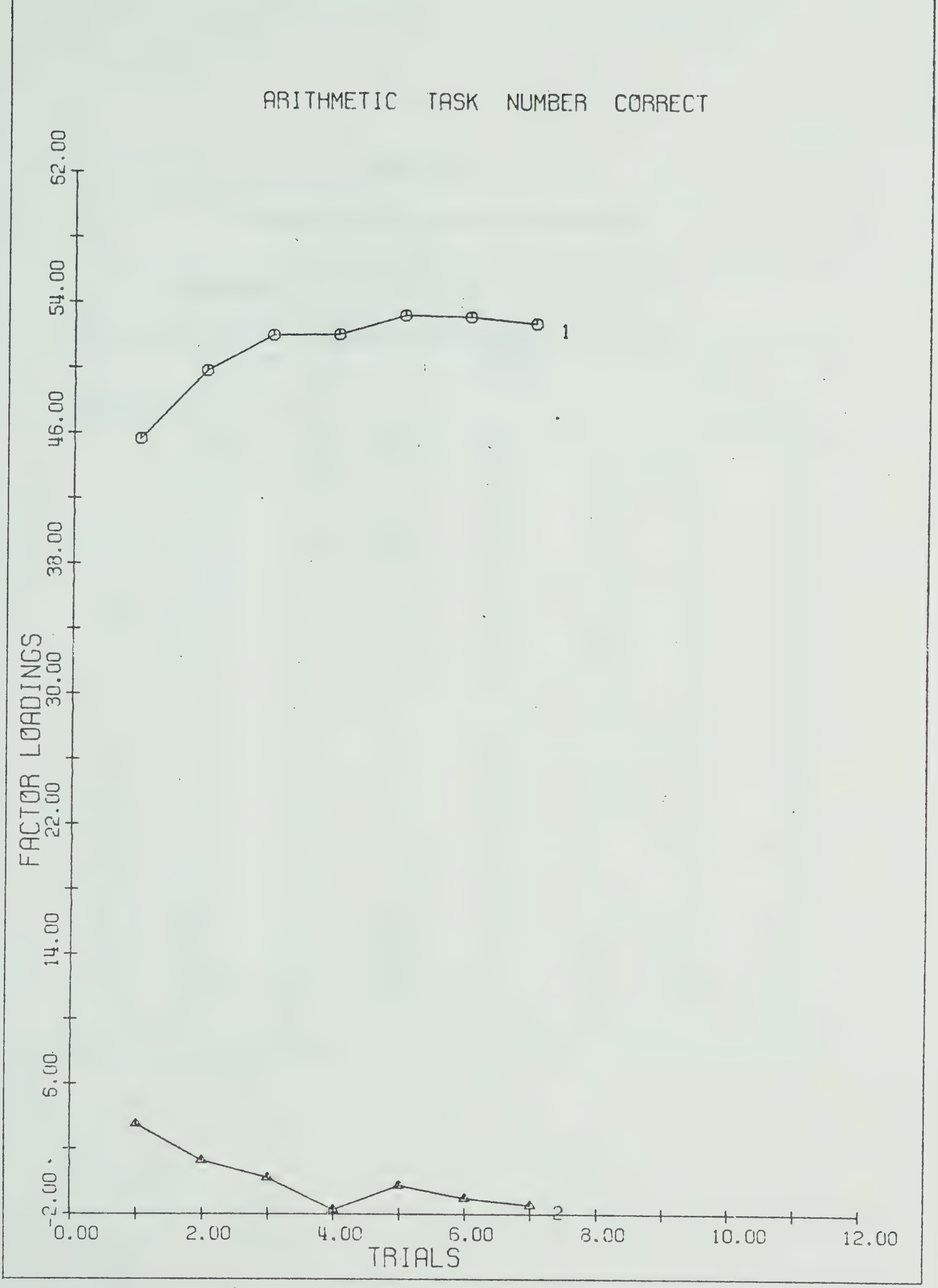
TABLE 4-4

CORRELATIONS

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

REFERENCE VARIABLES	COMPONENT		
	FIRST	SECOND	THIRD
RV1	.652*	.335	-.182
RV2	.278	.304	-.062
RV3	-.230	-.205	-.103
RV4	-.098	-.288	-.308
RV5	.340	.145	-.266
RV6	.640*	.144	-.453
RV7	.467*	.070	-.356
RV8	.095	.060	-.025
RV9	.462	.459	.007

*Significant at .05 level



ARITHMETIC TASK NUMBER CORRECT
FACTOR LOADINGS
TRIALS
62.00
54.00
46.00
38.00
30.00
22.00
14.00
6.00
-2.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
1
2

FIGURE 4-2

TABLE 4-5

APPROXIMATION MATRIX USING TWO COMPONENTS

$\hat{X}'_2$

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	34.2	38.6	40.8	41.8	42.0	42.3	42.2
2	42.2	49.6	53.3	56.1	55.3	56.2	56.4
3	55.7	57.7	58.9	56.6	59.7	58.7	57.7
4	35.1	36.0	36.5	34.8	37.0	36.2	35.5
5	38.1	47.3	51.9	56.3	54.2	55.8	56.4
6	42.1	47.1	49.6	50.5	51.0	51.3	51.1
7	44.5	47.4	49.0	48.1	49.9	49.5	48.9
8	56.7	58.2	59.2	56.5	59.9	58.7	57.6
9	47.9	53.5	56.3	57.3	57.9	58.1	57.9
10	43.5	49.8	53.0	54.8	54.7	55.3	55.3
11	48.0	53.2	55.9	56.6	57.4	57.6	57.3
12	53.7	54.3	54.8	51.5	55.3	53.8	52.7
13	43.5	47.3	49.3	49.2	50.5	50.3	49.9
14	40.7	46.3	49.2	50.7	50.7	51.2	51.1
15	47.4	53.1	56.0	57.2	57.6	58.0	57.8
16	53.1	55.4	56.7	54.8	57.6	56.7	55.9
17	52.2	55.6	57.5	56.5	58.6	58.1	57.4
18	36.7	40.5	42.5	42.8	43.6	43.6	43.4

TABLE 4-6

RESIDUAL MATRIX AFTER TWO COMPONENTS

$(X-\hat{X}_2)'$

ARITHMETIC TASK VARIABLE ONE - NUMBER CORRECT

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	-1.2	3.4	-4.8	3.2	3.0	4.7	-8.2
2	-0.2	-1.6	1.7	-1.1	1.7	-0.2	-0.4
3	1.3	-0.7	-2.9	1.4	0.3	0.3	0.3
4	0.9	-2.0	-3.5	-0.8	5.0	0.8	-0.5
5	0.9	-0.3	-0.9	0.7	-2.2	1.2	0.6
6	-0.1	2.9	-0.6	4.5	-3.0	-2.3	-1.1
7	-2.5	2.6	2.0	-1.1	0.1	2.5	-3.9
8	0.3	1.8	-1.2	2.5	-2.9	0.3	-0.6
9	0.1	1.5	-0.3	0.7	-2.9	-0.1	1.1
10	-1.5	2.2	-2.0	-5.8	1.3	2.7	2.7
11	2.0	-5.2	2.1	0.4	1.6	-2.6	1.7
12	0.3	-1.3	2.2	-0.5	-2.3	2.2	-0.7
13	-2.5	2.7	5.7	0.8	-1.5	-5.3	0.1
14	4.3	-8.3	1.8	2.3	-1.7	2.8	-1.1
15	-2.4	3.9	-0.0	-0.2	2.4	-4.0	0.2
16	-0.1	-2.4	2.3	-3.8	1.4	0.3	2.1
17	0.8	0.4	-2.5	0.5	0.4	-2.1	2.6
18	-0.7	0.5	-0.5	-3.8	1.4	0.4	2.6

The decisions as to the number of components to retain for the attention task variables were therefore much simpler than the decisions for the arithmetic variables. The components corresponding to the largest roots were checked for significant correlations with the arithmetic components. The jackknife groupings were then followed in selecting the number of components to retain, i.e. all components in a group would be retained if any were related to the Arithmetic Task components. The data matrices and the results of all calculations are given in Appendix G.

<u>The Data Means</u>

For each of the task variables, the first component curve approximated a curve through the data means. In fact, an analysis would show that in each case the first component curve was a linear function of a curve through the means. Thus there exist orthogonal transformations which would put the first component curves through the data means. Ross (1964) showed that putting the first component through the centroid of the person scores would result in the first component curve being proportional to the data means.

A rotation which accomplishes this was derived for each of the arithmetic task variables. The rotated component scores were compared to the reference variables to see if the rotation had made the components (essentially the components after the first) easier to identify.

Since the first component curve approximated the data means, it might be argued that the first component is superfluous. That is, since it represented merely an arbitrary scale value set by the researchers, it should not be considered as a component in the same sense as the second

TABLE 4-7

TASK VARIABLES

Arithmetic Task

1. Number Correct (NC):Two Components

2. Mean Latency (ML):Two Components

3. Linear Score (LS):Three Components

4. Quadratic Score (QS):Three Components

Attention Task

1. Mean Latency (M):One Component

2. Difference (D):Two Components

3. Median Latency (Md):Two Components

4. Standard Deviation (S):Two Components

or third components. Fortunately, it was possible to test the validity of the first components. This was done by removing the trial means from the original data matrices and re-analyzing the data. The variance/covariance matrix rather than the mean cross product matrix was used to compute the characteristic roots and vectors. The components extracted from the variance/covariance matrix were then compared to those extracted from the mean cross product matrix. If the data means produced an "extra" component, this should have been shown by the comparison.

Identification of the Components

Two approaches were taken in attempting to identify the components for the arithmetic task variables. The first consisted of rotating the Y matrix of component scores to a specific criterion and then comparing the rotated component scores with the reference variables.

One criterion for a rotation, rotating to the data means, has already been discussed. Although the object of this rotation is to match the first column of V to the trial means, the rotation is actually carried out by putting a component through the centroid of the person configuration. It is thus a rotation in the person space.

A second criterion for a rotation is that of simple structure in the person space. If there are groupings in the sample of students, such that one group of students load more heavily on one component than on other components and another group load more heavily on a different component, then an approximation to simple structure should be evidenced. A Varimax rotation was performed on the component score matrix Y for each of the arithmetic task variables. In addition, an Equamax rotation was

performed. Both the Varimax and Equamax rotations attempt to produce simple structure, but Equamax attempts to equalize the variance over the components.

All three of the rotations were in the person space. In each case, the efficacy of the rotation was determined by examining correlations between the rotated component scores and the scores on the reference variables. For the present study, the reference variables represented the only independent information which could be used in identifying the components. In many cases, apriori knowledge of the mathematical (or geometric) nature of the components could be used as criteria for rotations in the trial space, but in this study, it was felt that there was no such apriori knowledge.

The second approach to the identification of the components involves performing a principal components factor analysis. The Y matrix of component scores was combined with the scores on the reference variables. A correlation matrix was then formed and factored. The factor loadings obtained were compared to determine the relations between the components and the reference variables. The factors were rotated using the standard factor analysis rotations, Quartimax, Equamax, and Varimax, and the solution which most clearly illustrated the relationships was retained.

CHAPTER V

RESULTS

Analysis of Covariance

The means on the arithmetic pre-test for the experimental and control groups were, respectively, 125.44 and 136.55. Obviously, the randomization did not completely eliminate the initial group differences in arithmetic ability. As discussed earlier, the analysis of covariance adjusts the dependent variable for linear differences on a concomitant variable.

The table below gives the results of the one-way analysis of covariance on the arithmetic post-test scores.

Score	DF	MS	f	P
Group	1	2013.4375	5.456	0.026
Within	33	369.0378		

The adjusted group means for the post-test were 126.13 for the experimental group and 111.09 for the control group. The difference between the means was significant at a level of .026. (Less time was given on the post-test. Thus, in general, the scores were lower than the pre-test scores.)

The post-test, of course, measured only the area of arithmetic that was covered on the drill sessions. Nothing can be deduced about possible transfer effects to arithmetic performance in general. But the hypothesis tested was that some type of learning took place during the drill sessions. Since the post-test scores for the drill group were

significantly greater than those for the control group, this hypothesis was confirmed.

The Scoring Systems

The number of components judged to be significant for each of the arithmetic task variables is given in Table 4-7. As expected, the two scoring methods which combined latency with correctness yielded more components (three) than the methods which used number correct (two) and mean latency (two). The correlations between the scores on these various arithmetic components appear in the upper section of Table 5-1. These correlations give an indication of the relationships among the components. Based on the extremely high correlations between the first components for Number Correct (NC1), Linear Score (LS1), and Quadratic Score (QS1), it appeared that these three components were virtually identical. The first component for Mean Latency (ML1) was significantly (but to a lesser degree) correlated with the other first components.

The middle section of Table 5-1 contains the correlations between the arithmetic components and the attention components. In the lower section of the table are the correlations between the arithmetic components and the reference variables. A check of these correlations confirmed that there was little difference between NC1, LS1, and QS1.

The second component for NC was very similar to the second components for LS and QS but it was virtually unrelated to the second component for ML. The second ML component was significantly correlated with two of the QS and LS components, but had no significant correlation with either NC component.

TABLE 5-1

CORRELATIONS

	NUMBER CORRECT		MEAN LATENCY		LINEAR SCORE			QUADRATIC SCORE		
	NC1	NC2	ML1	ML2	LS1	LS2	LS3	QS1	QS2	QS3
NC1	1.000	.113	-0.644*	-0.360	0.904*	-0.046	0.401	0.890*	0.404	0.264
NC2	.113	1.000	-0.117	0.192	0.116	-0.797*	0.190	0.118	0.728*	-0.338
ML1	-.644*	-.117	1.000	0.582 *	-0.816	-0.111	-0.490*	-0.833*	-0.295	-0.461
ML2	-.360	.192	0.582*	1.000	-0.484*	-0.546*	-0.455	-0.479*	0.117	-0.662*
LS1	.904*	.116	-0.816*	-0.484*	1.000	0.022	0.462	0.942*	0.382	0.364
LS2	-.046	-.797*	-0.111	-0.546*	0.022	1.000	-0.000	0.024	-0.744*	0.570*
LS3	.401	.190	-0.490*	-0.455	0.462	-0.000	1.000	0.456	0.551*	0.746*
QS1	.890*	.118	-0.833*	-0.479*	0.942*	0.024	0.456	1.000	0.376	0.363
QS2	.404	.728*	-0.295	0.117	0.382	-0.744*	0.551*	0.376	1.000	-0.021
QS3	.264	-.338	-0.461*	-0.662*	0.364	0.570*	0.746*	0.363	-0.021	1.000
M	-.216	-.251	0.264	0.157	-0.254	0.056	-0.058	-0.255	-0.116	0.009
D1	.142	-.530*	-0.239	-0.600*	0.196	0.602*	0.332	0.191	-0.264	0.619*
D2	-.094	-.290	0.025	0.049	-0.072	0.125	-0.221	-0.073	-0.220	-0.084
Md1	.012	.042	-0.069	-0.217	0.035	0.041	0.191	0.031	0.076	0.178
Md2	-.149	-.479*	0.165	-0.194	-0.162	0.429	-0.076	-0.167	-0.385	0.172
S1	-.284	-.350	0.404	0.352	-0.355	0.038	-0.229	-0.357	-0.210	-0.131
S2	.101	.241	-0.184	-0.186	0.142	-0.151	0.200	0.141	0.247	0.032
RV1	.652*	.335	-0.571*	-0.103	0.667*	-0.350	0.477*	0.672*	0.660*	0.141
RV2	.275	.303	-0.136	-0.274	0.244	-0.172	0.236	0.222	0.269	0.055
RV3	-.229	-.204	0.036	0.070	-0.169	0.173	-0.203	-0.147	-0.278	-0.037
RV4	-.097	-.288	0.130	0.317	-0.121	0.064	-0.079	-0.108	-0.106	0.006
RV5	.399	.145	-0.229	0.007	0.316	-0.252	0.439	0.314	0.491*	0.211
RV6	.640*	.144	-0.379	-0.010	0.584*	-0.194	0.506*	0.581*	0.513*	0.292
RV7	.466*	.069	0.024	0.195	0.306	-0.279	0.217	0.275	0.407	-0.024
RV8	.094	.059	0.013	0.323	0.062	-0.045	-0.275	0.088	-0.086	-0.252
RV9	.461	.459	-0.251	0.032	0.409	-0.466*	0.319	0.407	0.610*	-0.029

* Significant at .05 level

All of this indicated that at least three components are necessary to completely describe the data. If the two NC components were used, the information contained in ML2 would be lost. Similarly, if the two ML components were used, the information contained in NC2 would be lost. In other words, if the arithmetic task was scored by counting the number of correct answers, the scores yield two components. If the mean latency was used, the scores again yielded two components, one of which was distinct from the two components for the number correct scores. Both weighting systems, linear and quadratic, resulted in scores which yielded three components and it appeared in each case that the three components contain all of the information that was contained by both the two components for ML and the two for NC.

Some mention should be made at this point about the correlations between two sets of component scores for the same task variable. The scores on ML1 and ML2, for instance, were significantly correlated. The intercorrelations were not zero as in factor analysis because of the presence of the data means. It will be seen later that when the first component by itself reproduces the data means the intercorrelations become zero. (They are also zero for the covariance solution.) But where each of the components reproduces part of the data means, the intercorrelations will not, in general, be zero.

The seven rows in the middle section of Table 5-1 contain the correlations with the attention task variables. The only attention component which consistently showed significant correlations with the arithmetic variables was the first Difference component (D-1). This com-

ponent correlated significantly with one arithmetic component for each of the four scoring methods. This consistency indicated that there was indeed a relationship between the attention task and the arithmetic task, and that in this instance the difference score was the most sensitive measure of the common phenomenon.

The lower section of the table gives the correlations with the reference variables. It should first be noted that there were no reference variables which showed a significant correlation with one of the components for NC or ML but not with one of the components for LS and QS. This tended to support the idea that either the linear scores or the quadratic scores contained by themselves all of the information contained by both the number correct and mean latency scores. The discussion that follows will concentrate on the identification of the components of the linear and quadratic scores.

The first component for both LS and QS was seen to correlate most highly (.67) with the score on the arithmetic pre-test, (RV1). It was also significantly (.58) correlated with the comprehension score on the reading test (RV6) and had a correlation of .41 with the digit span score, (RV9). This first unrotated component was thus related to some specific ability as measured by the three reference variables. The actual component curves are graphed in Figure 5-1 and Figure 5-2. Both of the first curves had a shape similar to that of the traditional learning curves. In fact if the average learning curves for these data were graphed, their shapes would be the same. The shape of the curve suggested that this first component accounted for most of the learning that

had occurred and, as might be expected, initial ability in arithmetic had something but not everything to do with the first component.

The second LS component and the third QS component were quite similar in several ways. First, they both showed a significant correlation with the attention task component and second, the shapes of these two curves were remarkably similar (after the first trial). LS2 had a significant correlation with the digit span but QS3 had no high correlations with any of the reference variables. These components seemed to represent the level of attention. The shape of LS2 was interesting in that it rose from trial one (on a Monday) to trial four (on Thursday) and then dropped for trials five and six (Friday and Monday) before the curve rose again for trial seven (on Wednesday). This "weekend effect" seemed to give additional support for the identification of the component as relating to attention.

The remaining curves, LS3 and QS2, showed significant correlations with arithmetic pre-test, digit span, comprehension and vocabulary. The correlations, while similar, were not exactly the same for the two components. The curves were of no obvious aid in identifying these components. Based on the correlations, they appeared to be related to a more basic cognitive ability which effected the student's performance in arithmetic.

The Data Means

The object of much of the remainder of the analysis was to look for a method of clarifying the meaning of the components. But before the rotations are discussed, the solution with the data means removed will be examined. The correlations for the component scores from the covariance

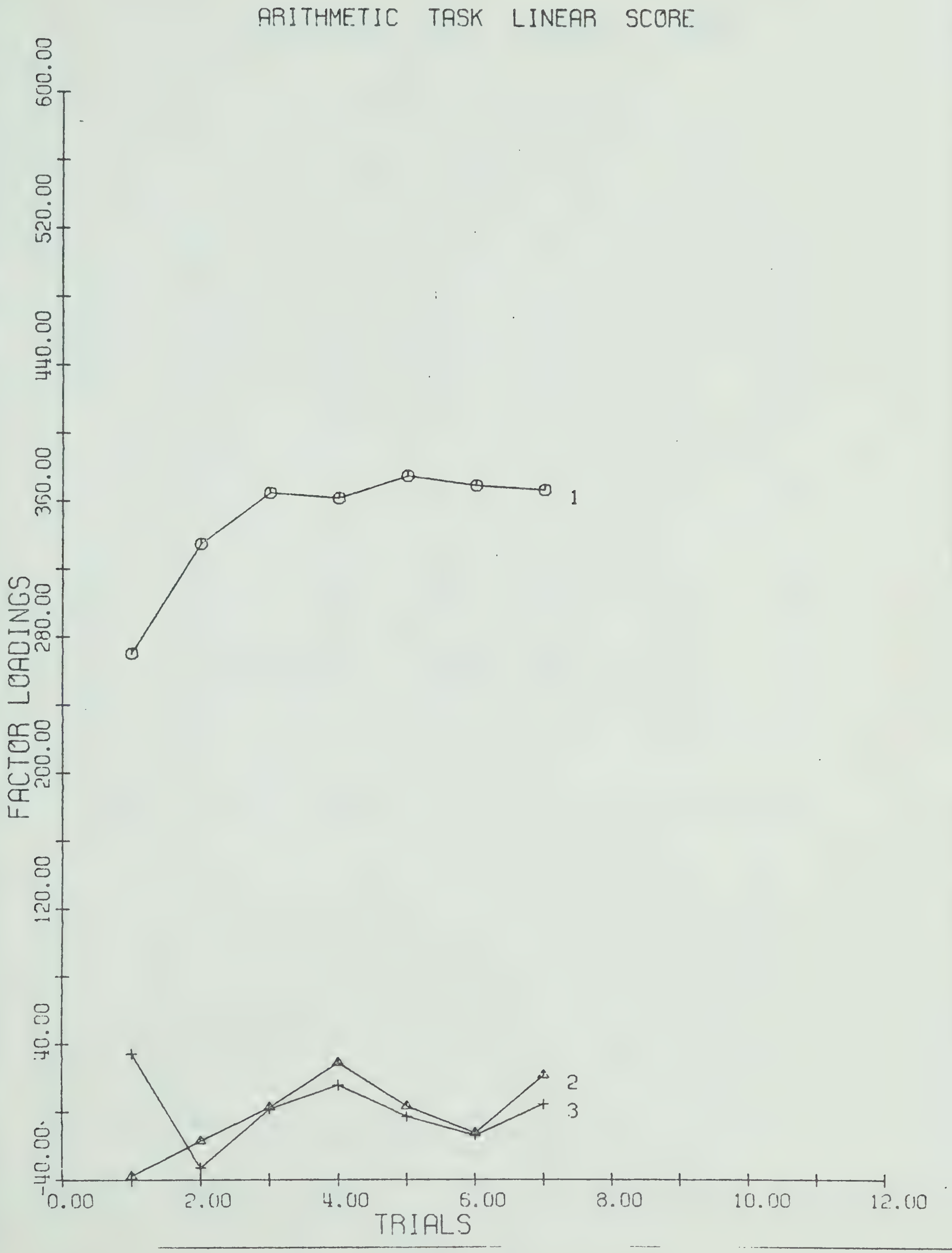

ARITHMETIC TASK LINEAR SCORE
FACTOR LOADINGS
600.00
520.00
440.00
360.00
280.00
200.00
120.00
40.00
-40.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TRIALS
1
2
3

FIGURE 5-1

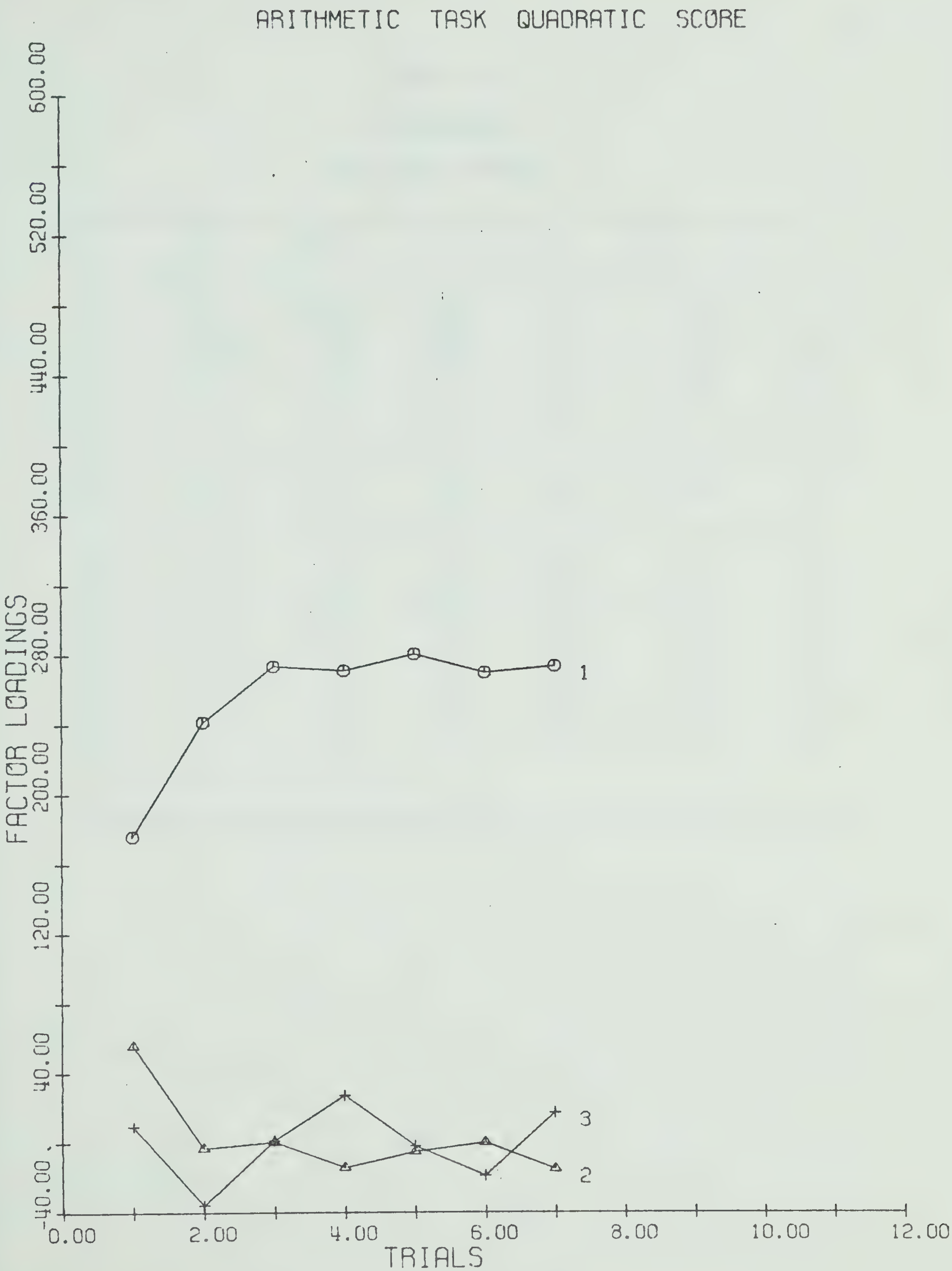

ARITHMETIC TASK QUADRATIC SCORE
FACTOR LOADINGS
600.00
520.00
440.00
360.00
280.00
200.00
120.00
40.00
-40.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
TRIALS
1
3
2

FIGURE 5-2

TABLE 5-2

CORRELATIONS

TRIAL MEANS REMOVED

	LS1	LS2	LS3	QS1	QS2	QS3
LS1	1.000	-.000	-.000	.942*	0.003	.003
LS2	-.000	1.000	.000	.006	.931*	.087
LS3	-.000	.000	1.000	-.002	.087	.938*
QS1	.942*	.006	-.002	1.000	.000	-.000
QS2	-.003	.931*	.087	.000	1.000	-.000
QS3	.003	-.087	.938*	.000	-.000	1.000
D1	.195	-.594*	.338	.186	-.536*	.396
D2	-.078	-.078	-.208	-.076	-.100	-.187
RV1	.669*	.370	.159	.678*	.405	.114
RV2	.244	.181	.146	.223	.185	.114
RV3	-.171	-.181	-.115	-.152	-.207	-.095
RV4	-.121	-.068	-.000	-.108	-.078	.017
RV5	.320	.267	.346	.322	.304	.345
RV6	.588*	.214	.231	.588*	.224	.227
RV7	.306	.289	.071	.279	.325	.042
RV8	.059	.039	-.429	.084	-.008	-.442
RV9	.410	.479*	.121	.413	.487*	.078

* Significant at .05 level

TABLE 5-3

CORRELATIONS

TRIAL MEANS REMOVED

WITH TRIAL MEANS		LS1	LS2	LS3	QS1	QS2	QS3
	LS1	.944	.001	-.014	.942	-.004	-.011
	LS2	.024	-.944	.021	.017	-.929	.109
	LS3	.477	.032	.797	.472	.100	.795
	QS1	.942	-.001	-.019	.944	-.009	-.016
	QS2	.389	.764	.374	.390	.798	.306
	QS3	.377	-.545	.650	.372	-.492	.704

solution are given in Table 5-2. The correlations for LS1, LS2, and QS1 were virtually unchanged. The last two QS components were now almost identical to LS2 and LS3. In Table 5-3, the correlations between the component scores of the mean cross product solution with the covariance component scores are given. The diagonal elements of this matrix confirmed the relationships found in Table 5-2.

It is important to note that removing the trial means from the data had little effect on the first component. Thus the argument that the first component is produced by the data means is seen to be in error. While the data means inflated the relative importance of the first component (by increasing the size of the first characteristic root), they did not change the nature of that component. The curves for the Linear Score covariance solution are shown in Figure 5-3. Note that the first component no longer resembled a traditional learning curve although the component scores had the same rank order as the component scores derived from the cross product solution.

Removing the trial means had some effect on the lower order components. Both scoring methods resulted in essentially the same three components. The second components, both QS2 and LS2, were still related to the level of attention. The curves, although reversed, showed even more clearly the "weekend effect" mentioned earlier. The third components no longer showed any significant correlations with other variables. There was no basis for their identification.

In this study, the choice of which solution, the cross-product or the covariance, to employ was not crucial. However in going to a covari-

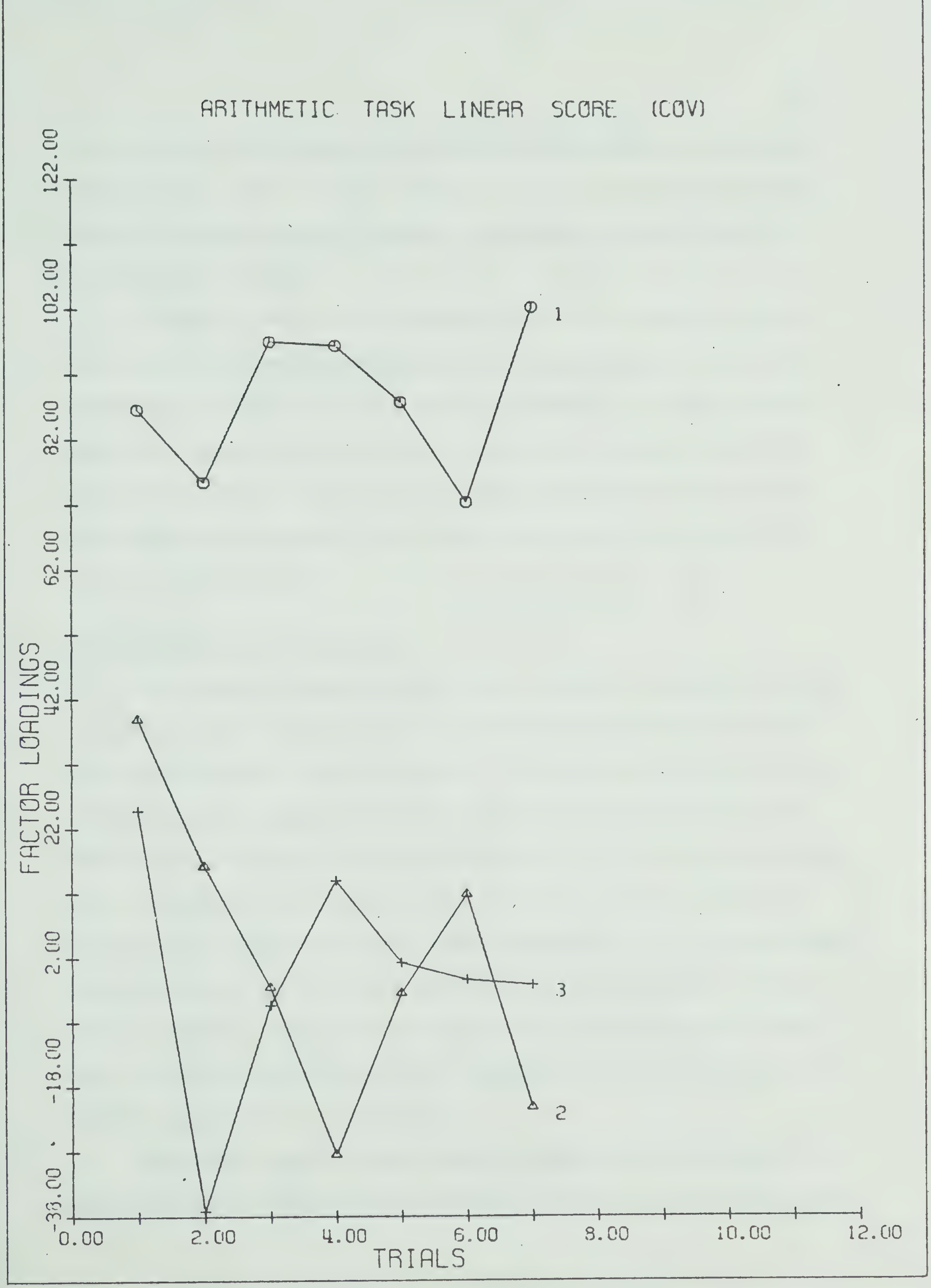

ARITHMETIC TASK LINEAR SCORE (COV)
FACTOR LOADINGS
TRIALS
122.00
102.00
82.00
62.00
42.00
22.00
2.00
-18.00
-38.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
1
2
3

FIGURE 5-3

ance solution, the information contained in the variance of the trial means is lost. Since for this study that variance represented learning and not a scoring artifact, a decision was made to proceed using the cross-product solution.

The results from the covariance solution did, however, make it clear that aside from the variance of the means, the linear score and quadratic score components are essentially identical. Since the means varied in a similar manner for both, there was no longer any need to retain both variables. The linear scoring method was simpler and for that reason the remainder of the analysis was carried out only on the linear score components.

Identification of the Components

The component scores obtained after the three rotations discussed in Chapter IV were compared with the scores on the reference variables. The correlations are given in Table 5-4. In no case did identification of the components appear to have been clarified. The decision as to whether or not to retain the rotated solution is, of course, an arbitrary one but one factor should be considered. After a rotation, the components are no longer arranged in order of magnitude. If a rotation does not significantly aid in the identification of the components, the unrotated components should be retained for the information they contain about the relative magnitude of the components. Thus for this study, the rotated components were not retained.

The second approach to the identification of the components is to perform a factor analysis on the unrotated component scores and the scores

TABLE 5-4

CORRELATIONS

LINEAR SCORES

	ROTATION TO MEANS			VARIMAX ROTATION			EQUAMAX ROTATION		
	LS1	LS2	LS3	LS1	LS2	LS3	LS1	LS2	LS3
LS1	1.000	.000	.000	1.000	-.308	-.293	1.000	-.293	-.283
LS2	.000	1.000	.000	-.308	1.000	-.665*	-.293	1.000	-.680*
LS3	.000	.000	1.000	-.293	-.665*	1.000	-.283	-.680*	1.000
D1	-.205	-.144	-.355	.502*	-.603*	.273	.501*	-.598*	.281
D2	.167	-.198	.160	-.156	-.117	.243	-.157	-.120	.242
RV1	.401	.249	.274	.402	.413	-.475*	.401	.421	-.470*
RV2	.243	.007	-.147	.188	.185	-.251	.188	.189	-.249
RV3	-.108	.244	.171	-.149	-.175	.243	-.150	-.178	.242
RV4	-.121	.006	.308	-.066	-.077	.082	-.066	-.078	.081
RV5	.196	.109	.048	.353	.247	-.440	.353	.254	-.436
RV6	.277	-.105	.305	.464	.236	-.390	.463	.245	-.384
RV7	.210	-.181	.175	.147	.307	-.310	.147	.311	-.308
RV8	-.053	-.047	.207	-.249	.103	.167	-.250	.099	.164
RV9	.265	-.073	.136	.194	.497*	-.505*	.194	.502*	-.504*

* Significant at .05 level

on the reference variables. The factors obtained are then rotated to a simple structure criterion. The factor loadings obtained for the components and reference variables after an Equamax rotation are given in Table 5-5. The unrotated loadings and the loadings after the Varimax rotation were similar to the results for the Equamax rotation.

Scores derived from the first component loaded most heavily on the first factor. This factor was related to the arithmetic pre-test, the reading test, IQ, and digit span. The identification given earlier, i.e. ability and understanding as relates to the specific arithmetic task, can not be improved upon.

The second factor was quite obviously a reflection/impulsivity factor, but it appeared to have little relation to any of the components.

The third factor seemed to involve the level of attention. The loadings on this factor for LS2 and D1 confirmed the relationship postulated earlier between the arithmetic and attention components. In addition, digit span loaded heavily on this factor, supporting its identification as an attention factor.

The last two factors gave some information about the identity of the third component. The fourth factor appeared to be age with vocabulary score loading heavily on it. The last factor was highly related to the second attention component and had a high loading for digit span. Again, the last component (LS3) which loads on both these factors as well as the first factor, seemed to represent a general ability component.

Individual Differences

The object in applying component curve analysis to learning data

TABLE 5-5

EQUAMAX FACTOR LOADINGS

LINEAR SCORES

	FACTOR					
	1	2	3	4	5	COMMUNALITIES
LS1	0.806	-0.219	0.264	-0.008	0.201	0.808
LS2	-0.103	0.075	0.880	-0.098	-0.133	0.818
LS3	0.421	-0.168	0.255	0.556	0.427	0.762
D1	0.117	0.161	0.800	0.396	0.020	0.836
D2	0.014	-0.120	0.109	0.081	-0.930	0.899
RV1	0.789	-0.083	-0.134	0.061	0.303	0.743
RV2	0.084	-0.888	-0.073	0.207	-0.147	0.865
RV3	-0.217	0.855	0.228	-0.140	0.047	0.851
RV4	0.086	0.967	0.017	0.062	-0.080	0.953
RV5	0.557	0.219	-0.278	0.610	0.101	0.818
RV6	0.856	0.023	-0.158	0.259	0.140	0.845
RV7	0.684	-0.116	-0.324	0.284	-0.287	0.748
RV8	0.274	0.271	-0.161	-0.819	0.211	0.889
RV9	0.518	-0.230	-0.454	0.301	0.411	0.786
CHAR. ROOTS	3.389	2.770	2.076	1.833	1.553	11.621

is to simplify the investigation of individual differences. For average learning curves to adequately represent the data, all students must have the same basic learning curve. In the component curve analysis which has been performed on the Linear Score variable, not only were three curves used to represent the data, but in addition a measure was provided which indicated those students whose scores were not fully accounted for by even three components. This measure was the error percentage. Table 5-6 shows the percentages for each of the students. From this table it can be seen that Person 1 and Person 18 have the highest error percentages. A check of the raw scores for these persons (given in Appendix F) showed that their scores followed a pattern quite different from that of the other scores. In each case, the scores dropped on the third trial and last trial. Person 18 is especially interesting in that he appeared to have reached a peak by the second trial and then stopped trying until the sixth trial (on Monday). The error percentages could be investigated further and all of the scores of those persons above the average error percentage could be examined to see in what way they differed from the others.

Possibly of even greater importance in examining individual differences are the component scores. These are given in Table 5-7. The scores for Person 1 and Person 18 were greatest for the attention component indicating that perhaps their erratic raw data scores were due to their inability to concentrate on the task.

Person 17 attained the highest raw score during the drill sessions. Her scores were seen to be greatest for the first component. The second

TABLE 5-6

ARITHMETIC TASK VARIABLE 3 - LINEAR SCORE

ERROR PERCENTAGES FOR 3 COMPONENTS
PERSON 1 : 17.63
PERSON 2 : 4.37
PERSON 3 : 4.69
PERSON 4 : 10.89
PERSON 5 : 2.29
PERSON 6 : 7.38
PERSON 7 : 8.20
PERSON 8 : 4.26
PERSON 9 : 3.97
PERSON 10 : 5.43
PERSON 11 : 2.62
PERSON 12 : 4.27
PERSON 13 : 8.18
PERSON 14 : 9.61
PERSON 15 : 3.55
PERSON 16 : 6.26
PERSON 17 : 3.94
PERSON 18 : 13.68
MEAN PERCENTAGE ERROR = 6.73

TABLE 5-7

COMPONENT SCORES

Y'

PERSON	COMPONENT 1	COMPONENT 2	COMPONENT 3
1	0.593	-0.000	-0.839
2	1.068	1.041	-0.068
3.	1.197	-0.984	0.992
4	0.490	-0.395	0.035
5	1.015	1.821	-0.168
6	0.950	1.027	-0.526
7	0.721	-1.285	-1.408
8	1.283	-1.440	0.451
9	1.111	0.149	0.405
10	0.947	0.609	-1.844
11	1.120	-0.036	1.279
12	1.065	1.185	0.069
13	0.838	1.185	-0.714
14	0.827	0.963	1.723
15	1.261	0.813	-0.686
16	1.143	-0.766	0.425
17	1.303	-0.153	0.624
18	0.492	-1.054	-2.023

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

component was more important in the scores for Person 8 who had high scores throughout the drill. Despite the similarity between their raw scores (and between their error percentages), these two students could be seen to differ in some aspect of learning. Just what this aspect may be was impossible to say due to the rather tentative identification of the first two components, but where the components can be more clearly identified, comparisons such as this should reveal much about individual differences.

CHAPTER VI

DISCUSSION

Computer-Assisted Drill

While the purpose of this study was not to judge the value of arithmetic drill on the computer, it was interesting to note that significant improvement was found for those students who worked on the drill program. The tests upon which this finding was based were constructed to measure only that area of arithmetic which appeared in the drill. Nothing can be said about any transfer effects to other areas of arithmetic performance or to general arithmetic ability. Yet after seven half-hour drill sessions, the students in the test group were able to solve more of the expressions in a given period of time than were their peers in the control group.

The drill program used in this study was constructed for research and not designed for teaching. A "real" drill program would take into account not only error rates but the nature of the errors made. For instance, problem numbers such as zero or unity could be programmed to appear more often until the student no longer missed any expressions containing those numbers. In the present study, boredom was welcomed and even predicted as a variable but for actual drill sessions, the objective should be to maintain a high level of attention. As attention wanes the stimuli could be varied or a completely different task could be initiated.

There is an important benefit of any computer-assisted learning

which this study illustrates. That is the ease with which the data can be analyzed. Figure 6-1 gives a sample of the print-out which was obtained for each student after each session. This information can be made available to the teacher as soon as a session has been completed. In actual practice, the data are usually stored either on disk or magnetic tape and the print-out is obtained later in the evening when the computer is no longer required for instructional purposes. Where a second computer is available, or if the central processor is sufficiently large, the tape can be processed at any time.

The analysis described in this paper was performed over the course of several months but there is no reason why the results of a component curve analysis, or any other sophisticated form of analysis, could not be made available immediately with virtually no human processing of the data.

While the data analyzed here has been from drill in arithmetic, the techniques described can be applied to any learning data (whether or not it is a result of computer-assisted instruction). Many basic learning tasks can be programmed onto the computer. Several discrimination learning tasks, for instance, have been programmed for the computer at the University of Alberta. Even pre-school children can use the terminals with no difficulty. The possibilities are extensive.

Component Curve Analysis

The technique of component curve analysis which was applied to the learning data in this study yielded extremely promising results. Despite the limited number of people and the relatively small number of trials, the components from the arithmetic task obtained were seen to be sig-

SAMPLE OUTPUT

ID NUMBER		DATE	TIME	
PROBLEM	ANSWER	LATENCY	NO. RIGHT	NO. WRONG
8 + 8 = **	14 X	4.5	0	1
6 + 2 = **	8	3.6	1	1
9 - 3 = **	6	3.0	2	1
4 - 1 = **	3	3.0	3	1
** + 3 = 9	6	2.0	4	1
** + 7 = 16	9	2.0	5	1
** - 4 = 3	7	1.0	6	1
** - 8 = 9	17	1.0	7	1

SUMMARY

RIGHT	WRONG	QUAD. SCORE	LIN. SCORE	MEAN LATENCY
7	1	57.3	73.3	2.5

FIGURE 6-1

nificantly correlated with reference variables. Perhaps even more important, the sets of components obtained from the different arithmetic task variables were seen to be surprisingly consistent. The analysis showed that by weighting the scores with the latencies, a more descriptive score can be obtained.

The results for the attention task were not as good. One of the components could be related to the arithmetic task, but the components from the various scoring methods were not as consistent as those from the arithmetic data. Part of this inconsistency may result from the nature of the task. The students for the most part found the task a pleasant diversion after the drill. Increasing the length of the attention task might result in a more sensitive measure of the level of attention. It is not surprising, though, that the difference score yielded the component which was seen to correlate with the arithmetic component for, unlike the median and mean score, a single difference score indicates the change in level of attention during a single performance of the task. It would be expected that however stimulating the attention task, a student who is becoming bored with working at the terminals will perform more poorly on the second half of the task than on the first.

Earlier some of the problems with component curve analysis were discussed. Of those, the most important is probably the problem of choosing the number of components. Because the magnitude of the characteristic values is determined by the size of the scores in the original matrix, there is no simple criterion such as stopping when the values are less than one. The jackknife technique used in this study to group

the characteristic values was found to be helpful but it does not provide the desired statistical test of significance. For studies similar to the present research, lack of such a test may not be very crucial. But research into more basic learning tasks requires a more reliable method for determining the number of components.

A computer program has been written at the University of Alberta which uses a Monte Carlo technique to create "instant" distributions for characteristic values. Knowing the distribution of the roots for randomized matrices should make it possible to determine which roots of the data matrix correspond to real components and which to random components. It is hoped that this work will result in the desired statistical procedure for determining the number of significant components.

The second problem relates to the data means. For this study removing the trial means did not significantly alter the solution. However there are cases where the trial means probably should be removed. For instance, if the number of problems given in each drill session had varied, then the variance of the trial means would have represented not only learning but an externally imposed constraint. In this case, a covariance solution would probably be more valid. For many situations, however, the trial means may represent an important part of the data and thus should be retained for the component curves to more accurately describe the learning that has occurred.

The major problem with rotations is that the simple structure criterion used in factor analysis has limited value. New rotations which are valid for learning data will have to be derived. If there is apriori

knowledge of the nature of the components, then a target matrix can be constructed for the trial loadings and a rotation based on a least squares fit obtained.

For the present study, none of the rotations improved the solution. The factor analysis was useful but no concrete identification could be made for the second component. Obviously the more reference variables and the more coherent the reference set, the better chance of identifying the components. Of course, the simpler the learning task analyzed, the more readily the components can be separated and identified. If this study were repeated, two changes would be made. First, a more coherent set of reference variables would be used and second, the task would consist of one type of problem which was equally unfamiliar to all of the students.

The identification of the components is crucial to the investigation of individual differences. It is a simple matter, if the components are known, to look at the magnitude and sign of the component scores to determine which components underlie each student's scores. The technique is limited only by the reliability of the learning data, the availability of suitable reference variables and criteria for rotation.

It is obvious, of course, that the technique of component curve analysis has not been developed to the same extent as factor analysis. But where learning data are involved, especially data from computer-assisted instruction, the technique may well provide the desired reduction in data while at the same time aiding in the investigation of individual differences.

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APPENDIX A

AUDIO SCRIPT FOR ARITHMETIC TASK

APPENDIX A

AUDIO SCRIPT FOR ARITHMETIC TASK

AA1: Hello. Today we are going to work on arithmetic. I will give you a series of arithmetic problems. They will be in the form two plus three is equal to five. Look at the sample problem on the screen.

(End of segment - Pause!)

Each time, one of the numbers will be missing. Your job is to supply the missing number. For instance, I might give you the problem two plus three is equal to blank. Look at the problem on the screen.

(End of segment - Pause!)

You would supply the missing number, in this case a five.

(End of segment - Pause!)

I might give you the problem in this form, two plus blank is equal to five.

(End of segment - Pause!)

You would supply the missing number, a three.

(End of segment - Pause!)

Or I might give you the problem blank plus three is equal to five.

(End of segment - Pause!)

You would, in this case, supply the two.

(End of segment - Pause!)

Watch the sign carefully, It may not always be a plus sign. Remember that you are to fill in the blank wherever it appears. On the top row of the keyboard in front of you, you will see numbers. Instead of writing down your answer to a problem, you will give me the number by pressing those keys. Note that the numbers are from zero to nine. If the answer to a problem is larger than nine, you will have to press two keys. For instance, if the answer to a problem is thirteen, you will press first a one and then a three. When you have pressed the number that you want, you must

then press the long space bar at the bottom of the keyboard. If you have answered correctly, your answer will appear in the blank. If you have not answered correctly, your answer will appear at the bottom of the screen. The correct answer will then appear in the blank. If you do not understand these instructions, raise your hand and someone will come to help you. If you do understand, press the space bar at the bottom of the keyboard and we will begin.

AA2: You will have to answer more quickly. Remember that once you have pressed the number that you want, you have to press the space bar at the bottom of the keyboard.

AA3: Good!

(End of segment - Pause!)

Correct!

(End of segment - Pause!)

Right!

(End of segment - Pause!)

You are doing very well!

(End of segment - Pause!)

Very good!

AA4: You have worked very hard. On the screen you can see how many problems you have done correctly. Goodby for now.

APPENDIX B

AUDIO SCRIPT FOR ATTENTION TASK

APPENDIX B

AUDIO SCRIPT FOR ATTENTION TASK

CC1: Hello! We are going to play a game with numbers. I want you to tell me, as quickly as possible, whenever a <u>number</u> appears on the screen in front of you.

Look at the keyboard beneath the screen. At the bottom of the keyboard is a long key. It is called a <u>space bar</u>. You can tell that a number has appeared on the screen by pressing the space bar.

Remember how the game works. You are to press the space bar as quickly as you can whenever you see a number on the screen. But you are to press the space bar <u>only</u> if a number appears.

If you understand how the game works, press the space bar and we will begin.

CC2: Wait for a moment and someone will come to help you.

CC3: That was very good. We are finished with this game. Rest for a moment and someone will show you what to do next. Thank you for playing. Goodbye for now.

CC4: No! That was not a number. Remember that you are to press the space bar only when a <u>number</u> appears on the screen.

APPENDIX C

SCORES ON REFERENCE VARIABLES

APPENDIX C

SCORES ON REFERENCE VARIABLES

PERSON	RV1	RV2	RV3	RV4	RV5	RV6	RV7	RV8	RV9
1	111	116	8	1	32	10	95	91	2.5
2	98	155	11	2	35	19	102[2]	95	2.1
3	163	188	6	1	47	31	109	93	6.5
4	56	64	20	3	31	7	83	93	2.2
5	93	79	16	3	34	21	106	94	1.7
6	67	116	9	1	40	26	96	99	3.6
7	130[1]	61	16	3	38	19	111	102	4.7
8	206	138	8	1	26	21	105	98	4.5
9	111	65	12	3	41	25	109[2]	94	4.8[3]
10	103[1]	135	13	2	28	13	109	93	3.4
11	228	79	12	3	45	32	125	97	3.6
12	136	222	8	1	37	24	113	95	3.6
13	77	123	10	1	19	11	97	100	1.9
14	115	130	11	2	35	24	101	96	3.3
15	158	48	20	3	31	20	83	103	2.1
16	146	86	17	3	44	27	101	95	2.9
17	220	98	10	2	44	30	112	98	6.6
18	75	59	12	3	34	23	105	101	2.8

1 Derived by Regression on RV5, RV6, and RV7.
2 Derived by Regression on RV5, RV6, and RV8.
3 Dervied from Median Scores of Experimental Group.

APPENDIX D

ARITHMETIC PRE-TEST AND POST-TEST

APPENDIX D

PART 1 - ARITHMETIC PRE-TEST

PART 1 - ARITHMETIC POST-TEST

3 + 4 = ________	4 + 4 = ________
2 + 4 = ________	8 + 9 = ________
2 + 3 = ________	2 + 0 = ________
3 + 8 = ________	6 + 9 = ________
6 + 4 = ________	3 + 1 = ________
3 + 6 = ________	9 + 7 = ________
3 + 5 = ________	0 + 2 = ________
6 + 8 = ________	6 + 7 = ________
9 + 0 = ________	8 + 0 = ________
2 + 2 = ________	0 + 3 = ________

5 + 0 = ________	3 + 0 = ________
1 + 3 = ________	6 + 6 = ________
9 + 1 = ________	5 + 5 = ________
5 + 8 = ________	1 + 0 = ________
4 + 5 = ________	7 + 2 = ________
4 + 3 = ________	7 + 4 = ________
4 + 6 = ________	8 + 2 = ________
7 + 0 = ________	0 + 4 = ________
3 + 2 = ________	9 + 3 = ________
1 + 2 = ________	7 + 6 = ________

PART 2 - ARITHMETIC PRE-TEST

PART 2 - ARITHMETIC POST-TEST

8 + ________ = 11

9 + ________ = 10

2 + ________ = 9

9 + ________ = 14

2 + ________ = 2

0 + ________ = 4

3 + ________ = 5

2 + ________ = 10

5 + ________ = 10

4 + ________ = 12

5 + ________ = 6

7 + ________ = 11

0 + ________ = 7

4 + ________ = 6

5 + ________ = 13

7 + ________ = 10

6 + ________ = 7

8 + ________ = 13

4 + ________ = 9

7 + ________ = 12

1 + ________ = 3

6 + ________ = 9

8 + ________ = 16

6 + ________ = 15

7 + ________ = 13

4 + ________ = 10

9 + ________ = 16

3 + ________ = 11

8 + ________ = 15

1 + ________ = 2

3 + ________ = 10

7 + ________ = 8

6 + ________ = 6

9 + ________ = 13

0 + ________ = 5

8 + ________ = 9

3 + ________ = 3

5 + ________ = 9

2 + ________ = 4

3 + ________ = 7

PART 3 - ARITHMETIC PRE-TEST

PART 3 - ARITHMETIC POST-TEST

______ + 3 = 12		______ + 9 = 11
______ + 2 = 5		______ + 0 = 9
______ + 2 = 11		______ + 4 = 8
______ + 8 = 12		______ + 2 = 7
______ + 7 = 9		______ + 4 = 4
______ + 5 = 14		______ + 1 = 8
______ + 0 = 1		______ + 6 = 15
______ + 5 = 12		______ + 8 = 17
______ + 6 = 11		______ + 4 = 13
______ + 4 = 7		______ + 5 = 6
______ + 5 = 13		______ + 1 = 6
______ + 7 = 7		______ + 7 = 15
______ + 6 = 14		______ + 2 = 8
______ + 6 = 9		______ + 0 = 4
______ + 5 = 8		______ + 4 = 9
______ + 9 = 12		______ + 8 = 13
______ + 4 = 5		______ + 3 = 5
______ + 6 = 10		______ + 7 = 16
______ + 9 = 15		______ + 4 = 12
______ + 1 = 9		______ + 1 = 1

PART 4 - ARITHMETIC PRE-TEST

PART 4 - ARITHMETIC POST-TEST

8	-	4	=	________	0	-	0	=	________
14	-	5	=	________	9	-	3	=	________
6	-	6	=	________	13	-	5	=	________
15	-	7	=	________	10	-	4	=	________
4	-	1	=	________	3	-	1	=	________
2	-	2	=	________	15	-	6	=	________
7	-	3	=	________	11	-	2	=	________
10	-	9	=	________	6	-	4	=	________
10	-	5	=	________	10	-	3	=	________
8	-	0	=	________	13	-	7	=	________

6	-	1	=	________	14	-	9	=	________
7	-	0	=	________	6	-	3	=	________
7	-	5	=	________	10	-	6	=	________
13	-	4	=	________	9	-	2	=	________
4	-	4	=	________	4	-	3	=	________
9	-	7	=	________	12	-	8	=	________
7	-	4	=	________	9	-	1	=	________
5	-	2	=	________	7	-	7	=	________
8	-	8	=	________	11	-	4	=	________
14	-	7	=	________	12	-	4	=	________

PART 5 - ARITHMETIC PRE-TEST

PART 5 - ARITHMETIC POST-TEST

3 - ________ = 3	5 - ________ = 4
11 - ________ = 8	6 - ________ = 1
8 - ________ = 7	9 - ________ = 2
5 - ________ = 2	5 - ________ = 5
15 - ________ = 9	12 - ________ = 9
6 - ________ = 4	9 - ________ = 1
7 - ________ = 1	8 - ________ = 0
3 - ________ = 0	7 - ________ = 3
12 - ________ = 8	13 - ________ = 7
6 - ________ = 0	16 - ________ = 9
11 - ________ = 3	10 - ________ = 1
10 - ________ = 9	18 - ________ = 9
2 - ________ = 1	10 - ________ = 5
4 - ________ = 1	9 - ________ = 4
16 - ________ = 8	3 - ________ = 2
4 - ________ = 2	8 - ________ = 3
15 - ________ = 8	15 - ________ = 7
5 - ________ = 1	12 - ________ = 5
10 - ________ = 6	13 - ________ = 9
7 - ________ = 2	9 - ________ = 7

PART 6 - ARITHMETIC PRE-TEST

PART 6 - ARITHMETIC POST-TEST

__________ - 6 = 4

__________ - 0 = 1

__________ - 7 = 2

__________ - 0 = 6

__________ - 5 = 7

__________ - 0 = 9

__________ - 6 = 1

__________ - 4 = 6

__________ - 2 = 6

__________ - 8 = 7

__________ - 7 = 3

__________ - 4 = 7

__________ - 4 = 3

__________ - 3 = 0

__________ - 5 = 9

__________ - 8 = 3

__________ - 1 = 7

__________ - 5 = 3

__________ - 9 = 8

__________ - 6 = 6

__________ - 9 = 2

__________ - 1 = 1

__________ - 6 = 9

__________ - 1 = 4

__________ - 7 = 8

__________ - 9 = 7

__________ - 6 = 2

__________ - 9 = 1

__________ - 7 = 5

__________ - 4 = 5

__________ - 2 = 9

__________ - 1 = 5

__________ - 6 = 5

__________ - 3 = 7

__________ - 8 = 2

__________ - 0 = 3

__________ - 2 = 7

__________ - 8 = 5

__________ - 5 = 4

__________ - 2 = 0

PART 7 - ARITHMETIC POST-TEST

_______ + 3 = 3	7 + 8 = _______
16 - 8 = _______	_______ + 3 = 10
9 + _______ = 5	12 - _______ = 6
0 + 8 = _______	2 + _______ = 11
_______ - 2 = 8	14 - 6 = _______
7 - _______ = 4	_______ - 7 = 7
9 + 9 = _______	3 + _______ = 8
0 + _______ = 6	_______ + 1 = 5
_______ - 9 = 18	2 + 7 = _______
5 - 1 = _______	_______ - 5 = 6
16 - _______ = 7	4 - _______ = 3
_______ + 1 = 10	12 - 5 = _______
5 - _______ = 3	4 + _______ = 11
3 - 2 = _______	1 - _______ = 1
8 + 5 = _______	_______ + 8 = 14
1 + _______ = 1	_______ - 1 = 6
_______ - 0 = 7	9 - 4 = _______
_______ + 9 = 17	3 + 7 = _______
8 + _______ = 14	_______ - 4 = 9
13 - 8 = _______	7 - 7 = _______

APPENDIX E

DIGIT SPAN TEST

APPENDIX E

DIGIT SPAN TEST

Spans

A	B	A	B	A	B	A	B
4	8	5	6	1	2	5	4
9	1	3	9	5	8	2	9
2	5	7	1	3	4	7	3
7	3	9	8	7	6	1	8
6	9	2	3	4	1	3	5
		4	2	2	9	4	2
				9	5	8	7
						6	1

The A Spans are presented in one ear (randomly chosen) and the B Spans in the other.

Scoring System:

1) Two points are awarded for each number in its correct position

2) The numbers not in their correct position are searched and two points are given for each correct run of two numbers.

3) The total of these two scores is changed to a decimal by dividing by twice the length of the span.

Example:

Span Given = 4 9 2 7 6
Span Repeated = 4 2 7 9 6

1) The 4 and the 6 are correct - 4 points
2) The 2 and 7 are a correct run - 2 points
3) Scores is 6 : 10 = .6

The score on the digit span variable is the sum of the scores on all eight spans.

APPENDIX F

ARITHMETIC TASK DATA AND RESULTS

ARITHMETIC TASK VARIABLE TWO - MEAN LATENCY

ORIGINAL MATRIX
X'

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	10.30	10.30	8.90	10.10	9.20	10.50	9.80
2	9.00	8.00	6.40	8.10	8.20	8.30	6.70
3	8.40	8.20	8.10	7.00	6.70	8.60	8.20
4	10.40	8.70	10.40	8.40	10.10	8.80	8.70
5	9.50	8.10	7.80	7.60	6.70	9.80	7.50
6	10.70	7.30	7.30	6.70	7.00	8.30	8.50
7	11.80	10.30	10.70	12.90	11.40	10.20	13.10
8	7.20	6.50	6.50	6.80	6.40	6.00	6.00
9	8.40	9.20	7.50	7.70	9.30	8.10	8.30
10	12.60	8.70	9.10	9.30	9.10	9.50	10.00
11	7.70	7.30	7.30	8.00	8.00	8.70	8.60
12	9.90	8.20	7.10	8.70	7.30	6.80	9.10
13	12.70	9.80	8.60	8.90	9.50	9.70	9.20
14	10.80	11.30	10.40	8.90	9.00	9.70	9.10
15	7.80	5.90	5.80	4.90	5.10	5.50	4.90
16	9.40	9.00	6.40	7.90	7.00	7.00	8.20
17	7.00	6.30	5.80	5.00	4.90	4.90	4.40
18	12.70	8.70	13.70	14.80	14.00	9.40	13.20
TRIAL MEANS	9.8	8.4	8.2	8.4	8.3	8.3	8.5
STD. DEV.	1.8	1.4	2.0	2.3	2.2	1.6	2.2

ARITHMETIC TASK VARIABLE TWO - MEAN LATENCY

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	99.204	84.269	83.135	85.685	83.956	83.581	86.748
2	84.269	73.113	70.985	73.133	71.728	71.937	74.068
3	83.135	70.985	71.423	73.255	71.850	70.562	73.799
4	85.685	73.133	73.255	76.490	74.437	72.660	76.743
5	83.956	71.728	71.850	74.437	73.136	71.263	74.826
6	83.581	71.937	70.562	72.660	71.263	71.839	73.649
7	86.748	74.068	73.799	76.743	74.826	73.649	77.663

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	537.040	24	102	391.851*
2	2.977	22	80	3.802*
3	0.937	20	60	1.471
4	0.788	18	42	1.637
5	0.547	16	26	1.546
6	0.364	14	12	1.473
7	0.212	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.428	-0.330	-0.824	0.079	0.046	-0.137	-0.054
2	0.366	-0.513	0.400	0.019	0.638	0.188	0.029
3	0.363	0.214	-0.055	-0.716	-0.133	0.530	-0.091
4	0.375	0.468	0.107	0.139	0.143	-0.089	-0.706
5	0.367	0.354	0.111	-0.286	0.184	-0.649	0.436
6	0.363	-0.416	0.366	-0.005	-0.693	-0.247	-0.139
7	0.379	0.255	0.032	0.545	-0.196	0.419	0.529

ARITHMETIC TASK VARIABLE TWO - MEAN LATENCY

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
536.56	52.24	$426.34 \leq \lambda_1 \leq 646.79$
2.89	1.64	$-0.57 \leq \lambda_2 \leq 6.35$
0.78	0.17	$0.43 \leq \lambda_3 \leq 1.13$
1.06	0.30	$0.42 \leq \lambda_4 \leq 1.69$
0.71	0.16	$0.38 \leq \lambda_5 \leq 1.05$
0.45	0.15	$0.14 \leq \lambda_6 \leq 0.75$
0.41	0.12	$0.16 \leq \lambda_7 \leq 0.66$

COMPONENTS

Group 1: 1

Group 2: 2

Group 3: 3, 4, 5, 6, 7

ARITHMETIC TASK VARIABLE TWO - MEAN LATENCY

TRIAL LOADINGS
B

TRIAL	COMPONENT 1	2	3	4	5	6	7
1	9.911	-0.569	-0.797	0.070	0.034	-0.083	-0.025
2	8.482	-0.885	0.387	0.017	0.472	0.113	0.013
3	8.412	0.369	-0.054	-0.636	-0.098	0.320	-0.042
4	8.696	0.808	0.103	0.283	0.106	-0.054	-0.325
5	8.513	0.611	0.107	-0.254	0.136	-0.391	0.201
6	8.421	-0.718	0.354	-0.004	-0.513	-0.149	-0.064
7	8.780	0.440	0.031	0.484	-0.145	0.253	0.243

COMPONENT SCORES
Y'

PERSON	COMPONENT 1	2	3	4	5	6	7
1	1.126	-0.383	1.446	0.577	-0.272	-0.181	-0.984
2	0.894	-0.436	0.475	0.142	0.361	-2.682	-1.263
3	0.900	-0.627	0.841	-0.271	-1.074	1.705	0.382
4	1.069	0.230	-0.149	-2.225	-0.135	-0.170	1.317
5	0.931	-1.075	0.378	-0.154	-1.860	0.095	-1.780
6	0.916	-0.802	-1.545	0.541	-1.345	0.252	1.395
7	1.312	1.324	0.624	1.584	-0.008	0.695	0.159
8	0.741	0.095	0.139	-0.434	0.579	-0.070	-0.990
9	0.953	-0.140	1.478	-0.288	1.119	-1.137	2.222
10	1.120	-0.290	-1.655	0.460	-0.840	-0.252	0.271
11	0.905	0.248	1.421	0.479	-1.433	-0.429	0.693
12	0.935	0.113	-0.773	1.649	1.124	0.951	-0.056
13	1.121	-0.890	-1.206	0.130	0.251	-1.380	0.441
14	1.128	-0.867	0.866	-1.359	1.005	1.569	-0.186
15	0.655	-0.752	-1.164	-0.765	0.294	0.110	-0.391
16	0.899	-0.575	-0.054	1.442	1.730	0.433	0.039
17	0.627	-0.661	-0.574	-1.031	1.254	0.515	-1.009
18	1.411	3.256	-0.766	-0.879	-0.133	-0.052	-0.733

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

ORIGINAL MATRIX
X'

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	124.77	214.87	182.27	248.73	266.40	259.70	143.47
2	240.30	319.03	422.50	387.23	406.23	394.63	425.16
3	402.33	406.96	398.50	439.20	465.36	418.06	416.03
4	152.20	166.83	122.13	171.43	218.57	194.83	175.67
5	200.43	307.13	353.46	417.76	386.90	373.56	419.73
6	205.87	353.83	343.53	416.86	340.13	324.13	329.20
7	184.53	294.10	296.00	212.63	271.63	316.53	187.37
8	426.16	470.53	450.63	453.53	441.53	470.46	449.13
9	311.70	365.63	410.46	425.53	364.46	417.10	424.73
10	168.13	346.06	328.06	304.33	377.33	389.33	380.73
11	346.50	333.23	434.16	409.16	429.06	376.43	417.53
12	342.30	366.20	428.90	335.33	383.03	424.80	338.00
13	156.40	304.56	377.80	321.73	299.77	256.17	316.90
14	233.03	153.60	302.30	351.23	309.16	345.46	318.20
15	293.90	452.90	444.20	471.53	497.20	430.36	482.83
16	342.26	349.50	461.56	352.36	449.10	429.46	415.56
17	389.20	433.03	433.53	470.76	492.16	462.80	512.53
18	106.17	236.60	145.90	94.00	169.90	252.83	195.40
TRIAL MEANS	257.0	326.4	352.0	349.1	364.9	363.1	352.7
STD. DEV.	98.7	87.3	103.0	104.7	91.1	77.7	108.0

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	75791	90253	98557	97430	101243	99882	98716
2	90253	114139	122071	120595	125601	124042	122463
3	98557	122071	134518	132029	136791	134789	133927
4	97430	120595	132029	132800	135890	133234	133187
5	101243	125601	136791	135890	141437	138856	137725
6	99882	124042	134789	133234	138856	137914	135410
7	98716	122463	133927	133187	137725	135410	136052

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	862234.250	24	102	351.780*
2	3199.727	22	80	1.612
3	2747.768	20	60	1.844
4	1440.297	18	42	1.109
5	1376.487	16	26	1.353
6	1090.014	14	12	1.660
7	562.876	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.291	-0.667	0.655	0.142	-0.044	0.133	-0.024
2	0.361	-0.304	-0.630	0.404	-0.412	0.211	-0.055
3	0.393	0.048	0.030	-0.726	-0.536	-0.164	0.032
4	0.390	0.513	0.302	0.477	-0.193	-0.392	-0.275
5	0.404	0.058	-0.054	0.092	0.279	-0.181	0.843
6	0.398	-0.222	-0.267	-0.203	0.608	-0.336	-0.444
7	0.395	0.380	0.088	-0.113	0.235	0.782	-0.108

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
861689	94811	$661639 \leq \lambda_1 \leq 1061739$
2313	578	$1094 \leq \lambda_2 \leq 3532$
2948	757	$1351 \leq \lambda_3 \leq 4546$
506	90	$316 \leq \lambda_4 \leq 696$
1988	336	$1278 \leq \lambda_5 \leq 2697$
2109	587	$872 \leq \lambda_6 \leq 3347$
1097	262	$546 \leq \lambda_7 \leq 1649$

COMPONENTS

Group 1: 1

Group 2: 2, 3

Group 3: 4, 5, 6, 7

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

TRIAL LOADINGS
B

TRIAL	COMPONENT 1	2	3	4	5	6	7
1	270.435	-37.746	34.348	5.397	-1.637	4.399	- 0.579
2	335.013	-17.175	-33.022	15.349	-15.291	6.954	- 1.298
3	365.135	2.691	1.553	-27.538	-19.900	- 5.429	0.751
4	362.103	29.010	15.821	18.101	- 7.173	-12.929	- 6.523
5	375.317	3.279	- 2.812	3.494	10.355	- 5.967	20.010
6	369.808	-12.574	-13.978	- 7.691	22.574	-11.084	-10.527
7	367.151	21.501	4.638	- 4.272	8.702	25.830	- 2.554

COMPONENT SCORES
Y'

PERSON	COMPONENT 1	2	3	4	5	6	7
1	0.593	-0.000	-0.839	1.232	0.703	-2.683	0.699
2	1.068	1.041	-0.068	-1.294	0.259	0.146	0.225
3	1.197	-0.984	0.992	1.408	-0.063	0.085	0.926
4	0.490	-0.395	0.035	1.136	1.257	0.022	0.964
5	1.015	1.821	-0.168	0.215	0.753	0.082	-0.421
6	0.950	1.027	-0.526	1.331	-1.359	-0.925	-0.865
7	0.721	-1.285	-1.408	-0.748	-0.455	-1.641	-0.051
8	1.283	-1.440	0.451	0.922	-0.736	0.539	-1.319
9	1.111	0.149	0.405	-0.042	-0.316	0.330	-2.316
10	0.947	0.609	-1.844	-0.423	1.257	0.640	0.345
11	1.120	-0.036	1.279	-0.517	-0.482	0.226	1.031
12	1.065	-1.608	0.069	-1.142	-0.439	-0.803	-0.373
13	0.838	1.185	-0.714	-0.929	-2.248	0.140	0.340
14	0.827	0.963	1.723	-0.894	1.818	-1.417	-1.175
15	1.261	0.813	-0.686	0.837	-0.410	0.613	1.217
16	1.143	-0.766	0.425	-1.826	0.250	0.154	1.422
17	1.303	-0.153	0.624	0.903	0.538	1.335	0.239
18	0.492	-1.054	-2.023	-0.206	1.306	1.225	-1.125

[illegible]

[illegible]

[illegible]

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

APPROXIMATION MATRIX USING 3 COMPONENTS

$\hat{X}_3'$

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	131.48	226.27	215.11	201.33	224.81	230.92	213.71
2	247.20	342.13	392.65	415.86	404.43	382.80	414.19
3	394.79	385.00	435.79	420.41	443.06	440.99	422.75
4	148.59	169.70	177.82	166.42	182.42	185.60	171.49
5	199.90	314.22	375.13	417.57	387.27	354.69	410.91
6	200.03	317.93	348.76	365.42	361.34	345.69	368.38
7	195.01	309.97	257.46	201.36	270.19	302.32	230.39
8	416.84	439.65	465.29	429.93	475.53	486.26	442.17
9	308.61	356.14	406.55	412.89	416.18	403.18	412.85
10	169.85	367.77	344.66	331.51	362.71	368.42	352.34
11	348.10	333.55	410.78	424.69	416.58	396.70	416.31
12	351.06	382.13	384.63	340.05	394.22	413.09	356.73
13	157.31	283.90	307.98	326.44	320.33	304.90	329.76
14	246.38	203.45	307.06	354.47	308.52	269.46	332.14
15	286.72	431.10	461.51	469.30	477.82	465.65	477.24
16	352.66	382.11	416.02	398.46	425.36	426.46	405.23
17	379.47	418.42	476.17	477.08	486.62	474.91	477.84
18	103.38	249.75	173.70	115.61	186.92	223.50	148.63

ARITHMETIC TASK VARIABLE THREE - LINEAR SCORE

RESIDUAL MATRIX AFTER 3 COMPONENTS

$(X - \hat{X}_3)'$

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	- 6.71	-11.40	-32.84	47.40	41.59	28.78	-70.24
2	- 6.90	-23.10	29.85	-28.63	1.80	11.83	10.97
3	7.54	21.96	-37.29	18.79	22.30	-22.93	- 6.72
4	3.61	- 2.87	-55.69	4.99	36.15	9.23	4.18
5	0.53	- 7.09	-21.67	0.19	- 0.37	18.87	8.82
6	5.84	35.90	- 5.23	51.44	-21.21	-21.56	-39.18
7	-10.48	-15.87	38.54	11.27	1.44	14.21	-43.02
8	9.32	30.88	-14.66	23.60	-34.00	-15.80	6.96
9	3.09	9.49	3.91	12.34	-51.72	13.92	11.88
10	- 1.72	-21.71	-16.60	-27.18	14.62	20.91	28.39
11	- 1.60	- 0.32	23.38	-15.53	12.48	-20.27	1.21
12	- 8.76	-15.93	44.27	- 4.72	-11.19	11.71	-18.73
13	- 0.91	20.66	69.82	- 4.71	-20.56	-48.73	-12.86
14	-13.35	-49.85	- 4.76	- 3.24	0.64	76.00	-13.94
15	7.18	21.80	-17.31	2.23	19.38	-35.29	5.59
16	-10.40	-32.61	45.54	-46.10	23.74	3.00	10.33
17	9.73	14.61	-42.64	- 6.32	5.54	-12.11	34.69
18	2.79	-13.15	-27.80	-21.61	-17.02	29.33	46.77

ARITHMETIC TASK VARIABLE FOUR - QUADRATIC SCORE

ORIGINAL MATRIX
X'

PERSON	TRIAL 1	2	3	4	5	6	7
1	29.04	113.05	80.22	138.57	155.16	144.16	40.03
2	141.59	223.14	335.91	286.90	302.43	286.96	331.43
3	304.14	309.86	296.89	346.31	373.44	316.23	314.76
4	37.92	59.66	19.03	73.54	115.87	91.55	86.16
5	102.64	207.87	261.72	322.62	291.12	267.00	316.23
6	95.18	248.30	239.99	324.46	243.07	220.12	223.37
7	88.55	201.49	187.55	110.71	169.12	205.42	79.89
8	327.72	374.50	359.41	351.80	350.13	377.98	357.57
9	209.12	261.53	312.96	326.80	249.51	305.78	315.19
10	49.94	238.37	216.37	195.67	269.37	276.71	273.39
11	246.83	235.23	334.38	302.05	320.24	265.92	308.48
12	228.68	265.84	335.14	245.42	285.51	323.78	229.78
13	42.70	204.68	270.50	217.67	191.06	149.37	208.68
14	126.74	59.26	202.95	248.60	207.33	234.24	218.56
15	202.61	367.77	361.86	396.33	416.28	345.75	404.83
16	241.83	251.19	368.84	255.71	349.24	332.07	309.86
17	302.33	352.43	353.11	393.56	415.29	386.05	439.91
18	- 5.22	135.31	42.40	8.09	66.11	141.79	97.07
TRIAL MEANS	154.0	228.3	254.4	252.5	265.0	259.5	253.1
STD.DEV.	103.0	90.4	108.0	107.5	97.1	83.7	111.9

ARITHMETIC TASK VARIABLE FOUR - QUADRATIC SCORE

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	34331	42206	48152	47362	49246	47484	47743
2	42206	60297	66000	65029	67885	65649	65786
3	48152	66000	76385	74191	76721	73973	74915
4	47362	65029	74191	75298	76291	72858	74696
5	49246	67885	76721	76291	79661	76183	77041
6	47484	65649	73973	72858	76183	74336	73930
7	47743	65786	74915	74696	77041	73930	76571

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	465908.313	24	102	180.458*
2	3606.148	22	80	1.780*
3	2789.332	20	60	1.828*
4	1560.080	18	42	1.206
5	1370.336	16	26	1.352
6	1076.107	14	12	1.616
7	570.817	12	0	0.0

*Significant for F.95

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.258	0.933	0.184	0.137	-0.036	0.092	-0.020
2	0.354	-0.049	-0.677	0.194	-0.493	0.364	-0.021
3	0.401	0.010	0.028	-0.907	-0.071	0.054	0.082
4	0.398	-0.236	0.519	0.187	-0.548	-0.312	-0.291
5	0.412	-0.082	-0.035	0.220	0.151	-0.345	0.795
6	0.397	0.005	-0.359	0.052	0.493	-0.443	-0.521
7	0.402	-0.254	0.328	0.185	0.430	0.666	-0.064

ARITHMETIC TASK VARIABLE FOUR - QUADRATIC SCORE

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
465341	70631	$316310 \leq \lambda_1 \leq 614372$
2975	611	$1686 \leq \lambda_2 \leq 4264$
2772	819	$1044 \leq \lambda_3 \leq 4499$
1105	220	$641 \leq \lambda_4 \leq 1569$
1707	370	$927 \leq \lambda_5 \leq 2487$
1937	527	$824 \leq \lambda_6 \leq 3050$
1046	280	$456 \leq \lambda_7 \leq 1636$

COMPONENTS

Group 1: 1

Group 2: 2, 3

Group 3: 4, 5, 6, 7

ARITHMETIC TASK VARIABLE FOUR - QUADRATIC SCORE

TRIAL LOADINGS
B

	COMPONENT						
TRIAL	1	2	3	4	5	6	7
1	176.229	56.033	9.697	5.410	- 1.322	3.030	- 0.487
2	241.815	- 2.915	-35.769	7.650	-18.249	11.939	- 0.506
3	274.014	0.624	1.466	-35.838	- 2.616	1.777	1.968
4	271.529	-14.171	27.392	7.379	-20.285	-10.240	- 6.943
5	281.137	- 4.927	- 1.873	8.694	5.572	-11.311	18.999
6	270.691	0.275	-18.950	2.068	18.252	-14.546	-12.448
7	274.321	-15.245	17.343	7.324	15.907	21.836	- 1.523

COMPONENT SCORES
Y'

	COMPONENT						
PERSON	1	2	3	4	5	6	7
1	0.399	-0.541	-0.781	0.711	-0.722	-2.617	0.380
2	1.078	-0.842	0.533	-1.153	0.903	0.366	0.274
3	1.247	1.349	0.199	1.370	-0.729	-0.324	0.971
4	0.273	-0.260	-0.065	1.506	0.735	-0.607	0.716
5	1.006	-1.510	0.954	0.350	0.268	-0.295	-0.270
6	0.901	-1.215	0.189	0.264	-2.147	-0.666	-0.726
7	0.581	0.257	-2.103	-0.900	-0.414	-1.192	-0.018
8	1.376	1.508	-0.599	0.511	-0.589	0.790	-1.194
9	1.103	0.158	0.462	-0.360	-0.374	0.537	-2.510
10	0.868	-1.651	-1.209	0.499	1.422	0.256	0.314
11	1.118	0.795	0.880	-0.658	-0.055	0.282	1.097
12	1.060	1.095	-0.991	-1.342	0.107	-0.904	-0.431
13	0.733	-1.442	-0.042	-1.790	-1.317	0.974	0.612
14	0.733	-0.210	1.857	-0.264	1.521	-1.927	-1.275
15	1.402	-0.898	-0.041	0.747	-0.654	0.642	1.164
16	1.174	0.852	-0.239	-1.352	1.372	-0.229	1.289
17	1.468	0.530	0.416	1.418	0.452	0.942	0.091
18	0.277	-0.705	-2.056	0.722	1.287	0.838	-1.218

APPENDIX G

ATTENTION TASK DATA AND RESULTS

ATTENTION TASK VARIABLE ONE - MEAN LATENCY

ORIGINAL MATRIX
X'

PERSON	TRIAL 1	2	3	4	5	6	7
1	1.50	2.51	1.23	1.49	1.39	1.28	2.80
2	1.57	1.12	1.32	1.06	1.13	0.94	1.89
3	1.30	1.41	1.53	1.32	1.47	1.46	0.92
4	1.37	1.53	1.34	1.01	1.08	1.00	0.96
5	1.33	2.09	1.49	2.17	2.89	3.26	2.58
6	1.10	0.82	0.86	0.90	0.87	0.85	0.81
7	1.04	0.85	0.87	0.90	0.79	1.28	1.02
8	1.68	1.76	1.37	1.74	0.95	1.08	1.02
9	1.31	1.42	1.86	1.33	1.35	1.56	1.34
10	0.84	1.61	1.17	0.92	0.93	0.97	0.90
11	0.84	1.01	1.28	1.02	0.86	1.34	1.43
12	1.74	1.03	1.15	2.74	1.39	1.42	2.65
13	1.40	1.23	1.30	1.16	1.25	1.60	1.66
14	1.36	1.36	1.43	1.00	1.56	1.06	2.44
15	1.02	1.01	1.22	1.01	0.91	0.96	0.99
16	1.20	1.12	1.26	1.22	1.11	1.23	0.94
17	1.24	1.24	1.05	0.96	0.91	0.98	1.08
18	1.52	2.86	1.54	1.73	1.93	1.80	1.98
TRIAL MEANS	1.30	1.44	1.29	1.32	1.27	1.34	1.52
STD. DEV.	0.25	0.54	0.23	0.49	0.49	0.53	0.67

ATTENTION TASK VARIABLE ONE - MEAN LATENCY

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	1.748	1.925	1.696	1.787	1.686	1.757	2.068
2	1.925	2.377	1.924	1.998	1.987	2.059	2.365
3	1.696	1.924	1.726	1.731	1.696	1.782	2.006
4	1.787	1.998	1.731	1.966	1.808	1.910	2.205
5	1.686	1.987	1.696	1.808	1.841	1.925	2.141
6	1.757	2.059	1.782	1.910	1.925	2.073	2.210
7	2.068	2.365	2.006	2.205	2.141	2.210	2.773

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	13.776	24	102	80.490*
2	0.237	22	80	1.757*
3	0.208	20	60	2.217*
4	0.160	18	42	3.040*
5	0.085	16	26	3.659*
6	0.021	14	12	1.041
7	0.017	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.348	-0.072	-0.442	0.375	-0.142	-0.536	-0.479
2	0.403	-0.450	-0.289	-0.624	0.388	0.080	-0.066
3	0.345	-0.366	-0.210	0.258	-0.552	0.438	0.374
4	0.369	0.193	-0.019	0.519	0.673	0.172	0.271
5	0.360	-0.053	0.424	-0.132	-0.115	-0.628	0.512
6	0.377	-0.147	0.690	0.083	-0.057	0.249	-0.537
7	0.435	0.773	-0.144	-0.330	-0.235	0.166	-0.041

ATTENTION TASK VARIABLE ONE - MEAN LATENCY

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
13.708	1.860	$9.785 \leq \lambda_1 \leq 17.632$
0.132	0.031	$0.066 \leq \lambda_2 \leq 0.198$
0.138	0.049	$0.035 \leq \lambda_3 \leq 0.241$
0.265	0.094	$0.067 \leq \lambda_4 \leq 0.463$
0.199	0.066	$0.059 \leq \lambda_5 \leq 0.339$
0.019	0.005	$0.009 \leq \lambda_6 \leq 0.029$
0.042	0.011	$0.019 \leq \lambda_7 \leq 0.065$

COMPONENTS

Group 1: 1

Group 2: 2, 3, 4, 5

Group 3: 6, 7

ATTENTION TASK VARIABLE ONE - MEAN LATENCY

TRIAL LOADINGS
B

	COMPONENT						
TRIAL	1	2	3	4	5	6	7
1	1.293	-0.035	-0.202	0.150	-0.041	-0.077	-0.062
2	1.495	-0.219	-0.132	-0.249	0.113	0.012	-0.009
3	1.281	-0.178	-0.096	0.103	-0.161	0.063	0.049
4	1.369	0.094	-0.009	0.207	0.196	0.025	0.035
5	1.337	-0.026	0.193	-0.053	-0.033	-0.090	0.067
6	1.400	-0.071	0.315	0.033	-0.017	0.036	-0.070
7	1.614	0.376	-0.066	-0.132	-0.068	0.024	-0.005

COMPONENT SCORES
Y'

	COMPONENT						
PERSON	1	2	3	4	5	6	7
1	1.269	1.030	-1.332	-2.291	0.675	0.709	-0.832
2	0.924	0.755	-1.011	0.214	-1.474	-1.070	-0.363
3	0.947	-1.261	0.368	0.779	-0.207	-0.655	1.123
4	0.839	-1.118	-0.745	0.131	-0.226	-0.845	-0.036
5	1.620	0.410	3.405	-0.642	0.477	-0.668	-0.025
6	0.627	-0.275	-0.183	0.697	-0.153	-1.344	-0.451
7	0.686	-0.089	0.361	0.542	-0.324	0.264	-2.367
8	0.971	-1.025	-1.254	1.041	1.548	-0.132	-0.544
9	1.029	-0.867	0.106	0.712	-1.111	1.514	1.034
10	0.748	-1.092	-0.366	-0.626	0.365	1.086	0.749
11	0.796	0.157	0.286	0.176	-0.883	2.764	-0.387
12	1.244	2.639	-0.383	1.976	1.721	0.309	0.877
13	0.980	0.156	0.271	0.288	-0.966	0.036	-1.818
14	1.053	1.247	-0.601	-0.937	-2.035	-0.927	1.502
15	0.721	-0.418	-0.249	0.561	-0.471	0.513	0.656
16	0.815	-0.675	0.089	0.888	-0.094	-0.188	0.181
17	0.758	-0.418	-0.526	0.163	-0.141	-0.619	-0.970
18	1.370	-0.951	-0.179	-1.699	1.445	-0.344	0.542

ATTENTION TASK VARIABLE TWO - DIFFERENCE

ORIGINAL MATRIX
X'

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	- 1.14	- 0.15	0.16	- 0.47	- 0.17	0.36	0.82
2	- 0.79	- 0.10	0.16	- 0.11	- 0.11	0.01	1.91
3	- 0.18	0.31	- 0.36	- 0.12	- 0.18	0.43	- 0.02
4	- 0.49	0.48	- 0.22	- 0.47	- 0.32	- 0.07	- 0.19
5	- 0.46	1.99	0.92	- 1.11	2.25	2.82	0.16
6	- 0.29	- 0.04	0.16	0.06	0.06	0.16	- 0.04
7	- 0.30	- 0.12	- 0.09	0.25	- 0.01	- 1.25	0.42
8	0.16	- 0.48	0.01	- 0.26	- 0.09	-0.42	0.06
9	- 0.25	0.63	- 1.31	- 0.49	0.62	- 0.79	- 0.14
10	0.02	1.37	- 0.16	0.11	0.0	0.24	0.06
11	- 0.03	0.19	0.73	0.0	0.23	0.35	0.48
12	0.23	- 0.07	- 0.40	- 1.93	- 0.41	- 1.09	2.02
13	- 0.34	0.05	- 0.08	0.05	- 0.02	- 0.78	0.79
14	0.09	0.49	0.41	0.33	0.99	0.13	- 1.30
15	0.25	- 0.09	- 0.20	- 0.19	- 0.09	0.09	0.12
16	- 0.11	- 0.15	- 0.04	0.23	- 0.13	- 0.01	- 0.02
17	- 0.29	0.05	- 0.18	- 0.10	0.05	0.05	- 0.01
18	- 0.07	- 1.10	- 1.11	- 1.39	- 1.67	- 0.57	1.22
TRIAL MEANS	-0.22	0.18	- 0.09	- 0.31	0.06	- 0.02	0.35
STD. DEV.	0.34	0.66	0.52	0.59	0.73	0.85	0.76

ATTENTION TASK VARIABLE TWO - DIFFERENCE

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	0.167	-0.059	-0.012	0.055	-0.037	-0.068	-0.137
2	-0.059	0.465	0.118	-0.036	0.398	0.368	-0.121
3	-0.012	0.118	0.279	0.119	0.220	0.281	-0.093
4	0.055	-0.036	0.119	0.442	0.050	-0.012	-0.363
5	-0.037	0.398	0.220	0.050	0.536	0.412	-0.227
6	-0.068	0.368	0.281	-0.012	0.412	0.722	-0.176
7	-0.137	-0.121	-0.093	-0.363	-0.227	-0.176	0.706

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	1.621	24	102	4.060*
2	0.898	22	80	4.091*
3	0.308	20	60	1.888*
4	0.226	18	42	1.994*
5	0.140	16	26	1.839
6	0.084	14	12	1.778
7	0.040	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	-0.018	-0.224	-0.129	0.300	-0.717	0.500	-0.281
2	0.421	0.249	-0.476	-0.245	0.203	0.597	0.277
3	0.279	0.014	0.570	-0.237	-0.430	0.025	0.597
4	0.146	-0.576	0.347	-0.461	0.282	0.304	-0.378
5	0.512	0.124	-0.253	-0.394	-0.351	-0.477	-0.390
6	0.567	0.285	0.403	0.548	0.222	0.083	-0.282
7	-0.375	0.677	0.292	-0.355	-0.092	0.257	-0.338

ATTENTION TASK VARIABLE TWO - DIFFERENCE

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
1.110	0.472	$0.113 \leq \lambda_1 \leq 2.107$
1.177	0.605	$-0.100 \leq \lambda_2 \leq 2.454$
0.244	0.082	$0.072 \leq \lambda_3 \leq 0.416$
0.341	0.107	$0.114 \leq \lambda_4 \leq 0.568$
0.213	0.057	$0.093 \leq \lambda_5 \leq 0.334$
0.130	0.044	$0.037 \leq \lambda_6 \leq 0.223$
0.103	0.031	$0.038 \leq \lambda_7 \leq 0.168$

COMPONENTS

Group 1: 1, 2

Group 2: 3, 4, 5, 6, 7

ATTENTION TASK VARIABLE TWO - DIFFERENCE

TRIAL LOADINGS
B

TRIAL	COMPONENT 1	2	3	4	5	6	7
1	-0.023	-0.213	-0.072	0.143	-0.268	0.145	-0.056
2	0.535	0.236	-0.264	-0.117	0.076	0.173	0.056
3	0.355	0.013	0.316	-0.112	-0.161	0.007	0.120
4	0.185	-0.546	0.192	-0.219	0.106	0.088	-0.076
5	0.652	0.118	-0.141	-0.187	-0.132	-0.138	-0.078
6	0.721	0.270	0.244	0.260	0.083	0.024	-0.057
7	-0.478	0.642	0.162	-0.169	-0.035	0.074	-0.068

COMPONENT SCORES
Y'

PERSON	COMPONENT 1	2	3	4	5	6	7
1	-0.202	1.190	1.035	-0.323	1.733	-1.651	1.192
2	-0.602	1.584	1.428	-1.745	0.828	0.205	-1.366
3	0.137	0.283	-0.285	0.682	1.264	0.574	-0.386
4	-0.040	0.326	-0.821	0.335	1.402	-0.032	2.616
5	2.851	2.578	-0.246	0.564	-0.410	-0.536	-0.392
6	0.140	0.051	0.371	-0.136	0.443	-0.596	0.439
7	-0.711	-0.191	-0.444	-2.071	-0.035	-0.481	0.584
8	-0.429	-0.101	-0.010	0.142	-0.953	-0.906	0.298
9	-0.192	0.247	-3.061	-0.676	0.940	-1.134	-1.612
10	0.519	0.402	-1.069	-0.490	1.098	3.086	0.738
11	0.330	0.546	0.995	-0.627	-0.804	0.550	0.727
12	-1.580	2.156	-1.148	-0.174	-2.231	0.343	0.950
13	-0.578	0.390	-0.155	-1.722	0.169	0.071	0.017
14	1.127	-0.847	-0.856	-0.420	-0.659	-1.199	1.231
15	-0.130	0.131	-0.135	0.577	-0.332	0.310	-0.864
16	-0.081	-0.188	0.298	-0.084	0.469	-0.067	-0.304
17	0.015	0.154	-0.214	0.001	0.699	-0.596	-0.023
18	-2.049	1.037	-0.056	2.241	0.693	-0.277	-0.112

ATTENTION TASK VARIABLE THREE - MEDIAN LATENCY

ORIGINAL MATRIX
X'

PERSON	TRIAL 1	2	3	4	5	6	7
1	1.00	1.35	1.10	1.20	1.30	1.05	1.70
2	1.15	1.05	1.30	0.95	1.10	0.75	0.90
3	1.20	1.40	1.45	1.20	1.10	1.25	0.80
4	1.20	1.30	1.20	0.90	0.75	0.90	0.90
5	1.00	1.10	1.05	1.35	1.85	1.40	1.80
6	1.00	0.80	0.80	0.85	0.85	0.80	0.80
7	0.90	0.70	0.80	0.80	0.80	0.75	0.75
8	1.40	1.55	1.20	1.40	0.90	0.90	0.95
9	1.10	1.15	1.05	0.95	1.05	1.25	0.95
10	0.80	0.90	1.10	0.80	0.90	0.90	0.80
11	0.80	0.90	0.90	0.90	0.80	1.15	1.20
12	1.30	0.90	1.00	1.55	1.35	0.90	1.45
13	1.40	1.20	1.25	1.10	1.25	1.20	1.70
14	1.10	1.00	1.20	0.85	1.05	0.70	1.40
15	0.95	0.95	1.00	0.90	0.90	0.95	0.90
16	1.10	1.10	1.30	1.25	0.95	1.20	0.90
17	1.10	1.25	1.00	0.90	0.90	0.95	1.10
18	1.15	1.35	0.80	0.95	1.10	1.30	0.95
TRIAL MEANS	1.09	1.11	1.08	1.04	1.05	1.02	1.11
STD. DEV.	0.17	0.22	0.18	0.22	0.26	0.21	0.34

ATTENTION TASK VARIABLE THREE - MEDIAN LATENCY

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	1.220	1.233	1.197	1.161	1.155	1.112	1.221
2	1.233	1.278	1.221	1.177	1.173	1.146	1.238
3	1.197	1.221	1.207	1.145	1.144	1.105	1.205
4	1.161	1.177	1.145	1.139	1.132	1.078	1.192
5	1.155	1.173	1.144	1.132	1.169	1.094	1.231
6	1.112	1.146	1.105	1.078	1.094	1.077	1.149
7	1.221	1.238	1.205	1.192	1.231	1.149	1.342

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	8.215	24	102	160.662*
2	0.109	22	80	3.664*
3	0.039	20	60	1.674
4	0.027	18	42	1.523
5	0.021	16	26	1.636
6	0.011	14	12	0.892
7	0.010	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.382	-0.289	-0.343	0.059	-0.241	-0.237	0.731
2	0.390	-0.409	0.097	-0.442	-0.412	0.488	-0.261
3	0.379	-0.347	-0.271	-0.075	0.776	-0.073	-0.224
4	0.369	-0.008	-0.141	0.685	-0.324	-0.187	-0.485
5	0.373	0.397	0.154	0.332	0.229	0.656	0.296
6	0.357	-0.041	0.828	-0.043	0.077	-0.414	0.080
7	0.395	0.685	-0.266	-0.464	-0.086	-0.252	-0.133

ATTENTION TASK VARIABLE THREE - MEDIAN LATENCY

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t = 2.110$
8.200	0.561	$7.016 \leq \lambda_1 \leq 9.383$
0.105	0.035	$0.031 \leq \lambda_2 \leq 0.179$
0.033	0.008	$0.015 \leq \lambda_3 \leq 0.050$
0.026	0.006	$0.013 \leq \lambda_4 \leq 0.038$
0.033	0.010	$0.013 \leq \lambda_5 \leq 0.054$
0.003	0.001	$0.001 \leq \lambda_6 \leq 0.004$
0.033	0.007	$0.019 \leq \lambda_7 \leq 0.047$

COMPONENTS

Group 1: 1

Group 2: 2

Group 3: 3, 4, 5

Group 4: 6, 7

ATTENTION TASK VARIABLE THREE - MEDIAN LATENCY

TRIAL LOADINGS
B

TRIAL	COMPONENT 1	2	3	4	5	6	7
1	1.095	-0.095	-0.068	0.010	-0.035	-0.025	0.074
2	1.117	-0.135	0.019	-0.073	-0.060	0.050	-0.026
3	1.085	-0.115	-0.053	-0.012	0.113	-0.008	-0.023
4	1.059	-0.003	-0.028	0.113	-0.047	-0.019	-0.049
5	1.068	0.131	0.030	0.055	0.033	0.068	0.030
6	1.024	-0.013	0.163	-0.007	0.011	-0.043	0.008
7	1.132	0.226	-0.052	-0.077	-0.012	-0.026	-0.013

COMPONENT SCORES
Y'

PERSON	COMPONENT 1	2	3	4	5	6	7
1	1.151	1.230	-0.316	-1.211	-0.695	1.034	-2.045
2	0.951	-0.598	-1.159	0.435	1.532	1.466	0.795
3	1.105	-1.508	0.778	0.634	1.023	0.688	-0.749
4	0.945	-1.284	-0.594	-1.115	-0.155	-0.128	0.067
5	1.259	2.413	1.290	0.830	0.405	1.460	-0.256
6	0.778	-0.144	-0.111	0.628	-0.266	-0.380	1.381
7	0.725	-0.089	-0.132	0.706	0.220	-0.527	1.037
8	1.096	-1.496	-1.127	0.516	-2.105	0.521	-1.156
9	0.987	-0.434	1.317	-0.106	0.152	-0.213	0.912
10	0.817	-0.359	0.369	0.021	1.629	0.347	-0.518
11	0.877	0.528	1.006	-0.859	0.075	-2.304	-1.175
12	1.115	1.181	-1.426	2.427	-1.085	-0.815	0.181
13	1.203	0.831	-0.631	-1.288	0.090	-1.438	1.348
14	0.967	0.599	-1.812	-1.304	1.045	0.259	0.421
15	0.863	-0.248	0.274	0.108	0.448	-0.315	0.085
16	1.026	-0.859	0.513	1.106	0.805	-1.479	-1.214
17	0.952	-0.335	-0.111	-1.199	-0.771	0.270	0.131
18	1.001	-0.408	1.921	-0.421	-1.728	0.911	1.494

ATTENTION TASK VARIABLE FOUR - STANDARD DEVIATION

ORIGINAL MATRIX
X'

	TRIAL						
PERSON	1	2	3	4	5	6	7
1	1.20	2.56	0.42	0.86	0.58	0.86	3.23
2	1.46	0.35	0.40	0.31	0.39	0.34	4.16
3	0.42	0.35	0.51	0.41	1.39	0.84	0.28
4	0.79	0.80	0.48	0.47	0.94	0.30	0.25
5	0.59	4.12	0.85	1.77	2.58	4.40	2.63
6	0.35	0.11	0.20	0.17	0.14	0.16	0.11
7	0.29	0.27	0.22	0.25	0.07	1.33	1.14
8	0.72	0.62	0.48	0.76	0.16	0.52	0.25
9	0.82	1.30	2.39	0.92	0.84	1.42	0.86
10	0.15	2.50	0.38	0.34	0.19	0.26	0.19
11	0.17	0.28	1.32	0.49	0.33	0.93	0.66
12	1.08	0.32	0.43	4.15	0.35	1.79	4.13
13	0.41	0.22	0.47	0.32	0.19	0.94	0.66
14	0.71	1.19	0.92	0.64	1.11	0.71	2.01
15	0.34	0.23	0.48	0.27	0.21	0.24	0.25
16	0.53	0.25	0.16	0.31	0.35	0.15	0.23
17	0.34	0.12	0.19	0.24	0.16	0.19	0.16
18	0.96	4.28	2.87	2.67	2.48	1.90	2.17
TRIAL MEANS	0.63	1.10	0.73	0.85	0.69	0.96	1.30
STD. DEV.	0.36	1.31	0.73	1.01	0.74	0.99	1.36

ATTENTION TASK VARIABLE FOUR - STANDARD DEVIATION

MEAN CROSS PRODUCT MATRIX
XX'/N

	1	2	3	4	5	6	7
1	0.525	0.805	0.516	0.700	0.500	0.654	1.206
2	0.805	2.940	1.327	1.479	1.529	1.906	2.022
3	0.516	1.327	1.069	0.904	0.831	0.978	1.070
4	0.700	1.479	0.904	1.740	0.912	1.397	1.958
5	0.500	1.529	0.831	0.912	1.033	1.201	1.187
6	0.654	1.906	0.978	1.397	1.201	1.908	1.851
7	1.206	2.022	1.070	1.958	1.187	1.851	3.542

CHARACTERISTIC ROOTS

COMPONENT	CHARACTERISTIC ROOT	D.F. NUM.	D.F. DEN.	MEAN SQUARE RATIO
1	9.773	24	102	13.918*
2	1.525	22	80	3.800*
3	0.535	20	60	1.734
4	0.445	18	42	2.164
5	0.300	16	26	2.708
6	0.121	14	12	1.768
7	0.059	12	0	0.0

*Significant for $F_{.95}$

CHARACTERISTIC VECTORS
V

	1	2	3	4	5	6	7
1	0.197	0.166	-0.127	-0.274	0.170	0.226	0.873
2	0.486	-0.543	-0.459	0.008	-0.479	-0.158	0.063
3	0.261	-0.237	0.231	-0.710	0.393	-0.358	-0.187
4	0.367	0.208	0.693	-0.091	-0.564	0.121	0.029
5	0.286	-0.282	0.026	0.015	0.276	0.829	-0.271
6	0.400	-0.165	0.313	0.642	0.426	-0.298	0.182
7	0.529	0.687	-0.376	0.015	0.102	-0.070	-0.302

ATTENTION TASK VARIABLE FOUR - STANDARD DEVIATION

CHARACTERISTIC ROOTS

BEST SINGLE VALUE	STANDARD DEVIATION	INTERVAL $t= 2.110$
9.305	3.253	$2.441 \leq \lambda_1 \leq 16.168$
1.397	0.580	$0.174 \leq \lambda_2 \leq 2.620$
0.341	0.132	$0.062 \leq \lambda_3 \leq 0.620$
0.530	0.172	$0.167 \leq \lambda_4 \leq 0.892$
0.796	0.247	$0.274 \leq \lambda_5 \leq 1.317$
0.268	0.079	$0.102 \leq \lambda_6 \leq 0.434$
0.121	0.037	$0.043 \leq \lambda_7 \leq 0.198$

COMPONENTS

Group 1: 1

Group 2: 2

Group 3: 3, 4, 5, 6

Group 4: 7

ATTENTION TASK VARIABLE FOUR - STANDARD DEVIATION

TRIAL LOADINGS
B

	COMPONENT						
TRIAL	1	2	3	4	5	6	7
1	0.617	0.204	-0.093	-0.183	0.093	0.079	0.212
2	1.518	-0.671	-0.335	0.006	-0.262	-0.055	0.015
3	0.816	-0.293	0.169	-0.474	0.215	-0.125	-0.045
4	1.147	0.256	0.507	-0.060	-0.309	0.042	0.007
5	0.895	-0.348	0.019	0.010	0.151	0.289	-0.066
6	1.251	-0.204	0.229	0.428	0.233	-0.104	0.044
7	1.655	0.848	-0.275	0.010	0.056	-0.025	-0.073

COMPONENT SCORES
Y'

	COMPONENT						
PERSON	1	2	3	4	5	6	7
1	1.320	0.648	-2.142	-0.113	-0.888	-0.520	0.740
2	1.000	2.196	-2.035	-0.635	1.353	0.284	-0.284
3	0.454	-0.401	0.522	0.078	1.175	2.268	-0.016
4	0.436	-0.375	-0.010	-0.574	0.160	1.753	1.599
5	2.201	-1.313	-0.121	3.012	0.579	0.108	-0.108
6	0.128	-0.003	0.111	-0.219	0.197	0.206	0.981
7	0.478	0.360	0.071	0.922	1.037	-1.275	0.476
8	0.395	-0.108	0.456	-0.393	-0.225	-0.156	2.374
9	0.966	-0.670	0.862	-1.587	1.575	-1.580	0.608
10	0.552	-1.068	-1.133	-0.222	-1.885	-1.115	0.683
11	0.483	-0.104	0.745	-0.622	1.264	-1.346	-0.770
12	1.602	2.597	2.333	0.360	-1.572	0.024	-0.054
13	0.386	0.120	0.311	0.213	0.893	-0.690	0.880
14	0.915	0.272	-0.665	-0.591	0.668	0.829	-0.981
15	0.221	-0.043	0.185	-0.443	0.310	-0.038	0.579
16	0.212	0.010	0.053	-0.270	0.077	0.833	1.321
17	0.150	0.024	0.157	-0.183	0.148	0.241	0.902
18	2.117	-1.469	0.367	-1.828	-1.003	0.507	-1.393